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Mission Statement

The International Baccalaureate Organization aims to develop inquiring, knowledgeable and caring young people who help to create a better and more peaceful world through intercultural understanding and respect.

To this end the IBO works with schools, governments and international organizations to develop challenging programmes of international education and rigorous assessment.

These programmes encourage students across the world to become active, compassionate and lifelong learners who understand that other people, with their differences, can also be right.

November 2002

La Déclaration de Mission de L'IBO

L'Organisation du Baccalauréat International (IBO) a pour but de développer chez les jeunes la curiosité intellectuelle, les connaissances et la sensibilité nécessaires pour contribuer à bâtir un monde meilleur et plus paisible, dans un esprit d'entente mutuelle et de respect interculturel.

À cette fin, l'IBO collabore avec des établissements scolaires, des gouvernements et des organisations internationales pour mettre au point des programmes d'éducation internationale stimulants et des méthodes d'évaluation rigoureuses.

Ces programmes encouragent les élèves de tout pays à apprendre activement tout au long de leur vie, à être empreints de compassion, et à comprendre que les autres, en étant différents, puissent aussi être dans le vrai.

Novembre 2002

Declaración de Principos de IBO

La Organización del Bachillerato Internacional tiene como meta formar jóvenes solidarios, informados y ávidos de conocimiento, capaces de contribuir a crear un mundo mejor y más pacífico, en el marco del entendimiento mutuo y el respeto intercultural.

En pos de este objetivo, la Organización del Bachillerato Internacional colabora con establecimientos escolares, gobiernos y organizaciones internacionales para crear y desarrollar programas de educación internacional exigentes y métodos de evaluación rigurosos.

Estos programas alientan a estudiantes del mundo entero a adoptar una actitud activa de aprendizaje durante toda su vida, a ser compasivos y a entender que otras personas, con sus diferencias, también pueden estar en lo cierto.

Noviembre 2002

Workbook contents

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Simon has a particular interest in the guidance and pastoral care of IB students transferring from the MYP to DP programme.

The IB learner profile

The aim of all IB programmes is to develop internationally minded people who, recognizing their common humanity and shared guardianship of the planet, help to create a better and more peaceful world.

IB learners strive to be:

Inquirers	They develop their natural curiosity. They acquire the skills necessary to conduct inquiry and research and show independence in learning. They actively enjoy learning and this love of learning will be sustained throughout their lives.
Knowledgeable	They explore concepts, ideas and issues that have local and global significance. In so doing, they acquire in-depth knowledge and develop understanding across a broad and balanced range of disciplines.
Thinkers	They exercise initiative in applying thinking skills critically and creatively to recognize and approach complex problems, and make reasoned, ethical decisions.
Communicators	They understand and express ideas and information confidently and creatively in more than one language and in a variety of modes of communication. They work effectively and willingly in collaboration with others.
Principled	They act with integrity and honesty, with a strong sense of fairness, justice and respect for the dignity of the individual, groups and communities. They take responsibility for their own actions and the consequences that accompany them.
Open-minded	They understand and appreciate their own cultures and personal histories, and are open to the perspectives, values and traditions of other individuals and communities. They are accustomed to seeking and evaluating a range of points of view, and are willing to grow from the experience.
Caring	They show empathy, compassion and respect towards the needs and feelings of others. They have a personal commitment to service, and act to make a positive difference to the lives of others and to the environment.
Risk-takers	They approach unfamiliar situations and uncertainty with courage and forethought, and have the independence of spirit to explore new roles, ideas and strategies. They are brave and articulate in defending their beliefs.
Balanced	They understand the importance of intellectual, physical and emotional balance to achieve personal well-being for themselves and others.
Reflective	They give thoughtful consideration to their own learning and experience. They are able to assess and understand their strengths and limitations in order to support their learning and personal development.

Bloom's taxonomy	DP group 4 assessment objectives	DP group 4 command terms (summary)
Knowledge Comprehension	Assessment objective 1	Define
		List
		Label
		State
Application Analysis	Assessment objective 2	Apply
		Describe
		Distinguish
		Outline
Synthesis Evaluation	Assessment objective 3	Analyse
		Compare
		Deduce
		Discuss
		Evaluate
		Explain
		To what extent

Table 1
The relationship between IB Diploma Programme command terms and Bloom's taxonomy

Research underpinning the command terms

Benjamin S Bloom, an educational psychologist concerned with the reliability of assessment items and practices, developed a framework for classifying educational objectives according to their cognitive complexity. His work, which is commonly known as Bloom's taxonomy, consists of six categories of the cognitive domain. The categories identified in Bloom's taxonomy were: ***knowledge***, ***comprehension***, ***application***, ***analysis***, ***synthesis*** and ***evaluation***. The taxonomy provided definitions for each cognitive process in every category and sub-category. Bloom proposed that the categories were hierarchical and ranged from the simple and concrete thinking process (lower-order thinking skills) to more complex and abstract functions of thought (higher-order thinking skills).

Bloom's original taxonomy was published under the title *Taxonomy of Educational Objectives: The Classification of Educational Goals, Handbook I: Cognitive Domain* (Bloom, Englehart, et al 1956).

Figure 1 shows a summary of Bloom's taxonomy categories for classifying objectives.

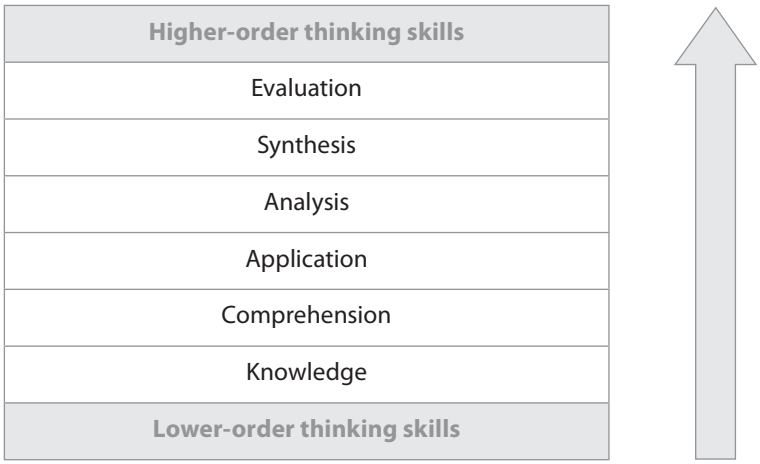

Figure 1
Bloom's taxonomy

IB assessment places an emphasis on the development of the "higher-order" cognitive skills (synthesis, reflection, evaluation and critical thinking) as well as more fundamental cognitive skills (knowledge, understanding and application). Understanding of and competency in the cognitive skills represented by the command terms supports the development of students as reflective thinkers. These are students who, in accordance with the *IB learner profile booklet* (2009), "exercise initiative in applying thinking skills critically and creatively to recognize and approach complex problems, and make reasoned, ethical decisions" and who would "give thoughtful consideration to their own learning and experience".

In the 50 years since its publication, the taxonomy has been subject to ongoing revisions (Anderson, Krathwohl, et al 2001) and criticisms (Marzano, Kendall 2007). However, the fundamental concept proposed by the taxonomy has been significant and influential in providing guidance for understanding, planning and developing educational objectives and assessment tools. Bloom's taxonomy, and the subsequent revised versions, offers a useful framework through which to express the diversity of the thinking skills required as part of teaching and learning.

It is worth considering similar concepts to Bloom's taxonomy that can influence the way in which schools look at the structures of learning. Säljö's (1979) and Bateson's (1972) work relates to levels of learning; whereas, both Bloom's and Biggs' (2003) work connects levels of understanding (addressing cognition and skills building) and are thus more directly relevant to the development of command terms.

Assessing the relevance of these similar concepts, revisions and the criticisms of Bloom's original taxonomy goes beyond the scope of this document. For those readers interested in these aspects the suggested reading list will provide a starting point.

Use of command terms in IB programmes

> What matters is not the absorption and regurgitation either of facts or of predigested interpretations of facts, but the development of powers of the mind or ways of thinking which can be applied to new situations and new presentations of facts as they arise.
>
> (Alec Peterson, first IB Director General, 2003: 47)

Command terms in the Diploma Programme

The phrase *command term* is used in the DP to refer to the words, generally verbs, specifically associated with the learning outcomes and assessment objectives of the programme. However, the use of the command terms is not exclusive to IB programmes. The thinking skills and cognitive processes represented by the command terms are an integral part of the daily communication that takes place between students and teachers during teaching and learning. Teachers use command terms when giving instructions, when questioning students, when posing problems and when eliciting responses from a class. Students are expected to understand and be able to respond effectively to the command terms present in teaching instructions, questions and problems presented to them.

Evidence of the use of command terms can be found in the objectives of the subject guides as well as in many examination questions in the DP. For example, students in the DP could be required to "**describe** the phenomenon of natural radioactive decay" or to "**discuss** the effectiveness of two strategies to reduce violence", or they could be asked to "**evaluate** the importance of Gandhi's leadership and methods in the struggle for Indian independence". To *describe*, to *discuss* and to *evaluate* are some examples from an extensive list of terms that the IB refers to as ***command terms***.

Students and teachers are expected to be confident using the command terms as part of teaching, learning and assessment. In an attempt to provide a working definition for the command terms, these could be defined as those *instructional terms that indicate the level of thinking and type of performance and/or behaviour that is required of students*.

Command terms in the Middle Years Programme

MYP subject guides make no explicit mention of the phrase "command term". However, the command terms are embedded in the objectives and assessment criteria of each subject area in the MYP. For example, in MYP sciences students are expected to "**apply** scientific knowledge and understanding to solve problems"; in MYP language A students "**compare** and **contrast** works, and connect themes across and within genres"; and in MYP arts students "**reflect** critically on their own artistic development and processes at different stages of their work".

The command terms and ATL

MYP approaches to learning (ATL) gives students the opportunity to develop a range of learning skills and strategies that will allow them to become more effective and reflective learners.

ATL skills areas include: **organization**, **collaboration**, **communication**, **information literacy**, **reflection**, **thinking** and **transfer skills**. Alongside the development of cognitive (thinking) skills, ATL promotes the development of **attitudes and dispositions** important for lifelong learning. It is through ATL that students are given the opportunity to reflect upon their own learning (metacognition), become aware of how they learn best, and consequently develop effective lifelong learning habits. Students are more likely to develop deeper conceptual understanding when they are aware of their own learning and can identify the type of thinking to draw upon in different contexts.

As teachers integrate ATL within subject content, they explicitly provide appropriate opportunities for the development of a range of learning skills and strategies, including those related to the use and application of the command terms. It is important that teachers make teaching and learning of command terms explicit to students when planning student learning expectations for ATL, as well as for other areas of interaction.

The outcome of using command terms is that students understand and know what to do when asked to "describe" as opposed to "discuss", or to "infer" as opposed to "explain". An understanding and mastery of the command terms is an ATL skill that can be applied in new situations across the MYP subject groups as well as in further courses, such as those of the DP.

Command terms across the continuum of IB programmes

In addition to supporting MYP teachers and coordinators, this document is helpful to DP teachers in understanding the skill development suggested by the command terms in the MYP. Likewise, the list of command terms illustrates students' skill development from the Primary Years Programme (PYP) to the MYP and then the DP.

Through their learning, over the course of the PYP, students acquire and apply a set of transdisciplinary skills: social skills, communication skills, thinking skills, research skills and self-management skills. These skills are valuable not only in the units of inquiry but also for any teaching and learning within the classroom and in life outside the school.

The thinking skills developed during the PYP can be demonstrated in many aspects of the MYP and DP, not least in the use of command terms. Command terms make thinking skills explicit by using them for questions in tests or essays; in formative and summative assessment; to help transfer interdisciplinary understandings; as part of an array of inclusive strategies; or as support for learners with differing language profiles.

The command terms in schools

> Teaching and learning are predominantly linguistics phenomena; that is we accomplish most of our learning through the vehicle of language … Therefore, language is a tool that teachers can use to enhance cognitive development. If we develop a successful programme for teaching thinking, we must also develop a language of cognition.
>
> (Costa, Marzano 2001: 379)

Below is a series of practical suggestions for schools in using command terms in teaching and learning.

Use precise terminology

Teaching and learning rely on the use of language. Therefore, it is important that teachers use precise terminology when explaining to students what is expected of them as part of an oral or written instruction. This point was stressed by Costa and Marzano (2001) who suggested that instead of asking students to "*Think what will happen if*" teachers should say "**Predict** what will happen if", or instead of saying "Look at these data" teachers should say "**Compare** these data". Other examples could include "Classify" instead of "Put into groups" or "Analyse" instead of "Let's work out this problem". Consistent and regular use of command terms across subject areas will help students to develop habits of mind, which will encourage the development of metacognitive awareness.

Make teaching and learning of command terms explicit

Schools should provide opportunities for the explicit explanation of command terms within the context of the subject groups. The teaching and learning of command terms should be embedded in the curriculum through ATL student learning expectations developed by schools.

By sharing command terms with students, teachers are able to give opportunities to practise relevant skills; to check understanding of the terms used to direct tasks; and to discuss what is expected or required, and the steps involved in completing tasks successfully. Each command term refers to specific thinking skills, practices and processes that constitute a subject or discipline, along with its content. In order to understand a discipline, which is a particular way of knowing, it is necessary to be fluent in the relevant command terms. The use of command terms overlaps between subject areas and should not be divided as being more or less applicable from one to another.

Ensure consistent use of command terms

The MYP command terms list presented in this document has been aligned with the command terms list used in the DP. Common or generic definitions have been provided for each command term. In some cases subject-specific clarifications have been included when a definition allows for subject-specific interpretation

(for example, "integrate"). It is important that both students and teachers share a common understanding of the command terms so that they can use them confidently and competently in teaching and learning. Teachers should use the command terms and their definitions in a consistent manner across the MYP and the DP.

Having a consistent definition of a command term enables those students with diverse learning needs to understand the meanings and their application across disciplines. This clarity of terminology allows these students to develop depth to their responses over time, which may reflect their true potential. Consistent application of command terms reduces stress and confusion about their meaning. Teachers can then focus on the specific skills of their discipline, which need to be taught in relation to the command terms of their subject area in order for students to successfully complete assessment tasks.

Schools are encouraged to make the command terms list available to both students and teachers to ensure that a common language and understanding of the command terms is developed within and across subject groups and programmes.

Support curriculum development and unit planning

The list of command term definitions aims to support teachers with the development of curriculum documents, including the formulation of student learning expectations for the areas of interaction, as well as the planning of individual units of work through the unit planning process. The use of command terms is instrumental during the process of developing interim objectives and interim assessment criteria.

Enable continuity in the development of thinking skills

While the definitions for the command terms remain the same, the expectation for the level of sophistication of students' understanding, responses and performances is expected to progress with students' maturity and development, and should correspond to the different stages of the MYP–DP educational continuum.

Opportunities to develop the thinking skills represented by the command terms should be sought out within and across the subject groups of the MYP. Collaborative planning should aim to support the transfer of thinking skills across different contexts and into new situations. Moreover, through vertical planning a developmental continuum of thinking skills could be planned to support students in their transition across programmes and to prepare them for success with their understanding of command terms in the DP. Figure 8 in *Making the PYP happen: A curriculum framework for international primary education* (December 2009) outlines the set of transdisciplinary skills that PYP students may acquire and apply. The thinking skills suggested include analysis, evaluation, metacognition and comprehension. These areas can be built upon and developed through the MYP.

Glossary of command terms in the MYP

The command terms listed are used to define the thinking skills that MYP students are expected to demonstrate. The definitions may vary when used in other contexts.

Command terms	MYP definitions
Analyse	Break down in order to bring out the essential elements or structure. To identify parts and relationships, and to interpret information to reach conclusions.
Annotate	Add brief notes to a diagram or graph.
Apply	Use knowledge and understanding in response to a given situation or real circumstances.
Appraise	Evaluate, judge or consider text or a piece of work.

Command terms	MYP definitions
Argue	Challenge or debate an issue or idea with the purpose of persuading or committing someone else to a particular stance or action.
Calculate	Obtain a numerical answer showing the relevant stages in the working.
Classify	Arrange or order by class or category.
Comment	Give a judgment based on a given statement or result of a calculation.
Compare	Give an account of the similarities between two (or more) items or situations, referring to both (all) of them throughout.
Compare and contrast	Give an account of the similarities and differences between two (or more) items or situations, referring to both (all) of them throughout.
Construct	Develop information in a diagrammatic or logical form.
Contrast	Give an account of the differences between two (or more) items or situations, referring to both (all) of them throughout.
Deduce	Reach a conclusion from the information given.
Define	Give the precise meaning of a word, phrase, concept or physical quantity.
Demonstrate	Prove or make clear by reasoning or evidence, illustrating with examples or practical application.
Derive	Manipulate a mathematical relationship to give a new equation or relationship.
Describe	Give a detailed account or picture of a situation, event, pattern or process.
Design	Produce a plan, simulation or model.
Determine	Obtain the only possible answer.
Discuss	Offer a considered and balanced review that includes a range of arguments, factors or hypotheses. Opinions or conclusions should be presented clearly and supported by appropriate evidence.
Distinguish	Make clear the differences between two or more concepts or items.
Document	Credit sources of information used by referencing (or citing) following one recognized referencing system. References should be included in the text and also at the end of the piece of work in a reference list or bibliography.
Estimate	Find an approximate value for an unknown quantity.
Evaluate	Assess the implications and limitations; make judgments about the ideas, works, solutions or methods in relation to selected criteria.
Examine	Consider an argument or concept in a way that uncovers the assumptions and interrelationships of the issue.
Exemplify	Represent with an example.
Explain	Give a detailed account including reasons or causes.
Explore	Undertake a systematic process of discovery.

Command terms	MYP definitions
Formulate	Express precisely and systematically the relevant concept(s) or argument(s).
Identify	Provide an answer from a number of possibilities. Recognize and state briefly a distinguishing fact or feature.
Infer	Deduce; reason from premises to a conclusion. Listen or read beyond what has been literally expressed.
Interpret	Use knowledge and understanding to recognize trends and draw conclusions from given information.
Investigate	Observe, study, or make a detailed and systematic examination, in order to establish facts and reach new conclusions.
Justify	Give valid reasons or evidence to support an answer or conclusion.
Label	Add title, labels or brief explanation(s) to a diagram or graph.
List	Give a sequence of brief answers with no explanation.
Measure	Find the value for a quantity.
Outline	Give a brief account.
Predict	Give an expected result of an upcoming action or event.
Present	Offer for display, observation, examination or consideration.
Prove	Use a sequence of logical steps to obtain the required result in a formal way.
Recall	Remember or recognize from prior learning experiences.
Reflect	Think about deeply; consider.
Recognize	Identify through patterns or features.
Show	Give the steps in a calculation or derivation.
Sketch	Represent by means of a diagram or graph (labelled as appropriate). The sketch should give a general idea of the required shape or relationship, and should include relevant features.
Solve	Obtain the answer(s) using appropriate methods.
State	Give a specific name, value or other brief answer without explanation or calculation.
Suggest	Propose a solution, hypothesis or other possible answer.
Summarize	Abstract a general theme or major point(s).
Synthesize	Combine different ideas in order to create new understanding.
Use	Apply knowledge or rules to put theory into practice.

Introduction

Holistic education has become a familiar topic within current education literature but there are conflicting opinions about what holistic education represents and a single definition remains elusive. There have been claims that holistic education reflects the education of the whole child but little clarity is offered to explain what this means, and in a field of education that is somewhat diverse, it is not surprising that there is confusion over what holistic education represents.

The purpose of this paper is to bring some clarity to what is meant by holistic education and to outline the characteristics and outcomes associated with it. This lack of clarity is an obstacle for teachers, parents and students alike and has the potential to obscure the advantages that this educational approach offers. Furthermore, such clarity would facilitate a comparison with other educational initiatives and allow curriculum designers to test their claims about whether they are delivering a programme of holistic education.

What is holistic education?

Holistic education does not exist in a single, consistent form. It is best described as a group of beliefs, feelings, principles and general ideas that share a family resemblance (Forbes 2003: 2). It is more than the education of the whole student and addresses the very broadest development of the whole person at the cognitive and affective levels. It emphasizes the education of the student beyond the confines of the classroom and moves the concept of a child-centred educational approach to a much more radical programme of education.

Holistic education focuses on the fullest possible development of the person, encouraging individuals to become the very best or finest that they can be and enabling them to experience all they can from life and reach their goals (Forbes 2003: 17). These experiences or achievements can be rare, special and deeply meaningful experiences for the individual or could represent a position, role or vocation that they perceive as unique or special and is an important goal in their life.

This could be described as the "vision" of holistic education. The aim of holistic education must be to prepare students for a fulfilling and productive life in which their skills and attributes are constantly challenged, developed and applied as part of their lifelong learning. It is an educational journey of personal discovery starting within formal education and then continuing throughout life. The learning and life experiences are continuous with individuals gaining in different ways from the various situations and demands that they are presented with. It could be argued that this is the aim of any good educational system. This is not disputed but while there remains a predominant focus on the value of a knowledge-based educational system the advantages of holistic education will not be realized.

What are the features of holistic education?

A programme of holistic education aims to encompass all aspects of personal learning and growth and emphasizes the development of active relationships at all levels, whether these are between the subject domains, between individuals and their peer groups and communities or between the individual and the world around them. Miller (1991: 3) has proposed that education may be described as holistic when it exemplifies the following characteristics.

- Holistic education nurtures the broad development of the students and focuses on their intellectual, emotional, social, physical, creative or intuitive, aesthetic and spiritual potentials.

- It promotes the importance of relationships at all levels within a learning community in which the educator and student work together in an open and collaborative relationship.

- There is an emphasis on life experience and learning beyond the confines of the classroom and the formal educational environment towards education as growth, discovery and a broadening of horizons. It encourages a desire to elicit meaning and understanding and to engage with the world.

- The approach empowers learners to examine critically the cultural, moral and political contexts of their lives. It leads learners towards actively challenging and changing cultural values to meet human needs.

Even a brief comparison of these characteristics with the aims and objectives of the IB programmes will reveal common features. These characteristics can be applied within any of the IB programmes and complement the continuum of IB programmes.

Holistic education has the capacity to lead the students into new areas of thinking, to broaden their personal and critical thinking and develop an appreciation of the world around them, and to realize the importance that relationships have within all these considerations. Importantly, holistic education has the capacity to empower students to think differently, to think creatively and reflect on their own values.

Undoubtedly, teachers would encourage their students to develop into well-educated, informed and participating members of society. This aim is accommodated within holistic education and offers teachers a framework within which to work. It also seeks to develop students actively beyond academic excellence alone.

Towards a profile of a holistically educated student – the student profile

Since a single definition of holistic education is elusive, the challenge is to identify what the outcomes of a holistic approach to education represent. In this way, teachers, parents and students can identify what is being aimed for and recognize the benefits that this approach can bring.

Holistic education can be associated with a number of recurring themes and values: the family resemblances that were referred to earlier (Forbes 2003: 2). These values are "guidelines for personal behaviour" (Thompson 1993) and it is these personal behaviours that characterize the outcomes of the holistic approach. The behaviours reflect a range of capabilities, skills and competencies that the students will begin to develop as they emerge from a formal programme of holistic education (Hare 2006). The behaviours and attributes associated with holistic education have been drawn together as outcomes within a **student profile**, the significant features of which are shown below.

- **Acts with social and academic maturity and integrity.**

  They are confident and at ease with individuals and groups with which they may be unfamiliar and show respect for the culture, opinions and values of others. They challenge accepted wisdom maturely and develop their own understanding from this experience. They learn from their errors, take responsibility for their actions and acknowledge the input and contributions of others.

- **Takes ownership of their own development and learning through planning and prioritization, and they ensure that through their own determination tasks are completed on time.**

  They take responsibility for their own personal and academic growth and the outcomes of this. They can set clear and realistic targets, prioritize conflicting demands and plan for success. They persist in their tasks and maintain high standards in their outcomes.

- **Demonstrates flexibility and a creative approach to problem solving.**

  Consistently, they can think creatively and laterally using approaches from a number of disciplines and experiences. They feel empowered and confident to find solutions and will take risks in new and unfamiliar areas of thinking and action.

- **Develops and maintains relationships through their interpersonal skills.**

  They act with care, consideration, compassion and empathy in their interactions with others from a variety of backgrounds. They learn from their interactions with others. Through active listening and open-mindedness they are ready to consider and accommodate views and opinions that challenge their own thinking.

- **Demonstrates a reflective approach and an attitude of continuous improvement.**

  They consider and review their work objectively and reflect on better ways of performing the task. They raise standards when the task is repeated so that improvements are made in the future.

- **Demonstrates effective written and oral communication skills.**

  They use the most appropriate way of delivering information in a variety of contexts and situations and appreciate the expectations and needs of the audience. They have sufficient confidence to change and adapt their style of communications should the situation or circumstances change.

- **Demonstrates good meeting management and involvement behaviours.**

  They contribute actively and collaboratively to support group discussions and meetings. They challenge others with respect and support and develop the ideas of others to encourage the decision-making process.

- **Seeks to bring clarity to decision-making.**

  They bring clarity to the plans and activities that are needed to complete any tasks. They consistently look for ways to exceed standards and expectations and learn from their actions for the future.

- **Through an understanding of their subject areas, their interdependencies and interrelationships, they can appreciate the interconnections in human knowledge; they can appreciate and debate global issues and the impact of human activity on the environment.**

  They develop a sound knowledge and understanding of the subjects that they study and can identify the features that draw the subjects and information together as a coherent whole. They are able to transfer skills between disciplines.

  From their interaction with others, their experiences and learning, they take a considered global perspective on international concerns and bring an informed appreciation of the issues relevant to these concerns.

- **Effectively uses the information resources that are made available to them to assist in the acquisition of further knowledge and its relevant application.**

  They can use data and information management methods effectively, including electronic data and library resources. They can evaluate effectively the value and content of data that they collect and use it appropriately.

  None of the skills and attributes that are associated with any one output exists in isolation. A brief inspection of the outcomes indicates that there are skills and competencies that are common within several of the outcomes. Furthermore, all of these outcomes can be regarded as interdependent and consequently, a focus on any single output will similarly address the development of others.

These outcomes should not be interpreted as representing definitive goals that must be achieved within the years of formal education. The skills and attributes supporting a holistic approach to education will develop at different rates and reflect the maturity of the individuals, their personal experiences and the relationships that they develop and from which they benefit. As the student matures, these skills and abilities will serve them well in understanding their role in society, the contribution that they can make and lead them to appreciate the potential that they have to offer.

Learning is envisaged as a lifelong experience. A major difference between the holistic approach and the familiar knowledge-based approach is that the former adopts a planned and considered approach to development beyond academic considerations and at the very broadest level. An emphasis is placed on relationships and the learning emerging from them and takes the student beyond the confines of a knowledge-based education. This delivers personal and interpersonal skills that will bring considerable advantages to the students' progress beyond their formal education.

The IB learner profile and the student profile

The IB learner profile outlines the attributes and aspirations of internationally minded students engaged in the IB programmes. In common with the student profile, the learner profile identifies educational outcomes at the academic, personal and interpersonal levels. A comparison of both profiles reveals striking similarities, for example, "communicators" within the learner profile has features in common with "acts with social and academic maturity" and "demonstrates good meeting management and involvement behaviours" within the student profile.

The comparison offers evidence that the outcomes of holistic education complement the educational aims within the continuum of the IB programmes and consequently all the IB programmes. Furthermore, since the student profile reflects the outcomes of a holistic approach to education, this comparison suggests that the IB programmes also support that holistic approach.

The implications of holistic education

Holistic education is a clear departure from the knowledge transmission approach to education that has been familiar in the past. Holistic education prepares a student for lifelong learning in which the educational focus moves towards the life skills, attitudes and personal awareness that the student will need in an increasingly complex world. The commitment to the subject knowledge of the student is maintained but is complemented by learning how to learn, the critical evaluation of knowledge gained and the use of this knowledge in a broad range of contexts; it is these that represent the education currency of the future.

Consequently there is an emphasis on a broad educational approach that addresses the intellectual, personal and interpersonal development of the student and puts in place many of the values, attitudes and skills that will serve the student well throughout life. Such a broad development cannot be claimed by the knowledge-laden education systems that have dominated for so many years. It brings closer the realization of "learning to be" and "learning to live together" that have been identified in the Delors report (UNESCO 1996: 14).

This broad development of the student must be carefully planned at the individual level and has obvious repercussions on teaching strategy, curriculum design and the assessment of progress. The evaluation of the "soft" personal skills such as integrity, care, consideration, negotiation and active listening has never been easy, but these are new challenges in educational assessment. The skills and attributes will constantly be in dynamic development as various situations and scenarios present themselves in which these skills are necessary and appropriate. Evaluative information must be sought on behaviours and actions from a number of sources and in a range of situations and these fed back to the student in a constructive and supportive manner. These behaviours lend themselves to peer review and informal assessment. This could be problematical and brings into focus the importance of the relationship of the teacher and student and between groups of students. Simply offering an assessment in terms of effort and a grade will no longer suffice; such "soft" skills cannot be graded on any scale.

Holistic education calls for students to take ownership of their own development. This ownership will take different forms at different ages and levels of student maturity. Nevertheless, it is a key element within holistic education and teachers will need to be at the forefront of driving this issue and encouraging this ownership.

The supportive role of the parent within holistic education is essential. The behaviours identified in the student profile will also manifest themselves outside the classroom and parents should recognize, encourage and praise such positive behavioural changes.

Holistic education is a radical endeavour. The educational journey starts the process of self-actualization and self-realization through relationships, and interconnectedness with other individuals, groups and the world around them is an integral part. Formal education is merely the starting point of this lifelong process.

Holistic education: Implications for the teacher

Holistic education represents a new journey for both student and teacher and one in which both parties will grow and critically examine perhaps strongly held values and beliefs. For the educator this could be an unsettling experience; the teacher is moving out of the comfort zone of subject specialization into areas of personal uncertainty. No longer is the teacher depending on subject expertise but is guiding students in developing and examining their own values and prejudices, their critical thinking and behaviours and confronting opinions that are new to them without clear demarcation of right or wrong. This is uncertain territory. Frequently this may be a joint journey of discovery for both student and teacher, with the teacher bringing their greater life experience to the learning process.

Holistic education challenges teachers to think differently about student cognitive and affective development and to examine critically how they practise their craft. The working relationship between the student and the teacher changes; it becomes more inclusive, dynamic and egalitarian. The appropriate pedagogical approach will become one of active, planned interventions that are developed for students to meet their development needs. The teacher's skills of facilitation, guidance and mentoring will feature strongly in promoting learning and understanding at both the academic and social levels. The aim is for students to understand, for example, the importance of relationships, the different ways of regarding knowledge and its evaluation, the importance of life skills and the impact that the students have on others around them.

Teachers must also examine the learning culture within their school so that it is conducive to creating an inclusive learning community that stimulates the growth of a person's creative and inquisitive engagement with the world. The aim becomes the development of healthy, curious individuals who can learn what they need to know and apply it in any new context in which they find themselves and who are self-motivated and confident learners.

Why is holistic education important and what advantages are there in understanding what it represents?

Holistic education broadens and deepens the educational process. It represents a planned approach that encourages personal responsibility, promotes a positive attitude to learning and develops social skills. These are essentials in the modern world in which we live.

The identification of the outcomes of holistic education has advantages. These outcomes clarify the purpose of this educational approach for students, teachers and parents. Furthermore, it allows parents and students to make an informed choice when they are confronted with different educational systems. The educational characteristics of the IB programmes have much in common

with the outcomes associated with a holistic approach to education. If parents favour a holistic approach to the education of their children then the IB programmes offer such an approach.

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John Hare
July 2010

Student Learning Outcome Areas	Student Expectations	Key Questions	Student Expectations	Key Questions
Awareness and understanding of:	Grades 7 and 8		Grades 9 and 10	
Innovation and Change	*Understanding the meaning of ingenious, the ways humans have created and why humans want and need to create.* <u>Students will demonstrate this by:</u> • Defining Human Ingenuity. • Recognizing milestones in human innovation. • Identifying historical reasons for different creations.	• Why do you humans create, develop or change products or solutions? • How do products, ideas or solutions change over time?	*Awareness of the systems, solutions and products created by humans and the processes related to their development.* <u>Students will demonstrate this by:</u> • Identifying various systems and processes, not simply products, as examples of human creation. • Recognizing the different values mankind places on various forms of creation.	• How do we place value on creation? • What are the various examples of human ingenuity in our lives?
Reflection on:	Grades 7 and 8		Grades 9 and 10	
Ingenuity	*Analyze the factors that have led to different innovations over time and the various effects that these innovations have had.* <u>Students will demonstrate this by:</u> • Identifying the cause relationships of various innovations. • Recognizing of examples of perpetual innovation.	• What future developments can I foresee? • How does innovation inspire innovation? • What impact have creations had on individuals, society and the world?	*Illustrate considered insight into how humans approach problem solving from a variety of schools of though and using different sets of skills. Reflect on the various implications of human ingenuity.* <u>Students will demonstrate this by:</u> • Outlining various religious, cultural, social, political, environmental, effects related to a human innovation. • Relating a variety ways in which a problem can be approached or solved.	• What are the consequences of human innovations? • How does using perspectives make my problem solving more effective?
Action:	Grades 7 and 8		Grades 9 and 10	
Creating Solutions	*Using creative approaches to develop ideas about and solutions for individual problems.* <u>Students will demonstrate this by:</u> • Outlining steps to solving a problem. • Analyzing the effectiveness of the approaches they have used.	• What are my challenges and what solutions can I come up with? • What steps do I take to solve a problem? • How do I tell if my chosen solution was the most effective?	*Using creative approaches to develop ideas about and solutions for group and community problems.* <u>Students will demonstrate this by:</u> • Demonstrating willingness to take action towards solving the problems of others as well as personal problems. • Developing and apply innovative techniques for making changes and creating solutions. • Collaborating in groups to develop effective solutions.	• How can I use my thinking in one subject to help me in another? • How can we benefit by sharing and comparing ideas about a problem? • Why should I care?

Student Learning Outcome Areas	Student Expectations	Key Questions	Student Expectations	Key Questions
Awareness and understanding of:	Grades 7 and 8		Grades 9 and 10	
Ourselves and Society	*A developing awareness of our human needs and systems including personal well being, relationships, lifestyles and group dynamics.* <u>Students will demonstrate this by:</u> • Recognize and identify current global health issues, global social concerns, and personal development matters. • Identify ways that current global health and social issues impact their individual lives.	• How do the global health and social issues impact my life? • What are some developmental matters I have to consider as I get older?	*A insightful awareness of our human needs and systems including personal well being, relationships, lifestyles and group dynamics.* <u>Students will demonstrate this by:</u> • Recognizing the causes and possible long term implications of current global health issues, global social concerns, and personal development matters. • Identifing measures being taken to alleviate global health and social problems throughout the world. • Identifing steps individuals can take in their own lives to address develpmetal matters.	• What are some of the factors that create global health and social issues? • What can I do as I grow?
Reflection on:	Grades 7 and 8		Grades 9 and 10	
Our Responsibilities	*Analyzing the ways in which our health and social choices impact us and others.* <u>Students will demonstrate this by:</u> • Distinguishing between beneficial and harmful choices. • Outlining positve and negative impacts of their social choices.	• What social and developmental responsibilities do I have?	*Analyzing personal health and social responsibilities and how they can influence systems and groups.* <u>Students will demonstrate this by:</u> • Identifing ways they can influence the choices of others. • Outlining their duties in society as a responsible citizen.	• What skills do I need to be a responsible citizen?
Actions:	Grades 7 and 8		Grades 9 and 10	
Our Choices	*Analysis of how we make choices regarding behaviour, ethics, values, wants and needs, and healthy living.* <u>Students will demonstrate this by:</u> • Identifying the causes that led to different choices they have made. • Predicting the long term implications that could result from their various choices.	• What social and developmental choices have I already made? • How do I know my choices are good ones? • Where can my choices take me?	*Analysis of our daily health, lifestyle and social choices; how they affect our personal development and impact the world around us.* <u>Students will demonstrate this by:</u> • Giving examples of the ways their choices are interconnected with the choices of others. • Identifying how their lifestyles and the lives of others have been influenced by their choices.	• Where can my choices take others? • How do I promote change?

ENVIRONMENTS

Student Learning Outcome Areas	Student Expectations	Key Questions	Student Expectations	Key Questions
Awareness and understanding of:	Grades 7 and 8		Grades 9 and 10	
A Variety of Environments	**Understanding what environments are, and how we are both a part of and dependent on them.** Students will demonstrate this by: • Classifying environments as natural, built or virtual. • Defining which environments we are a part of. • Outlining specific examples of how environments contribute to their lives and well being.	• How do I affect my environments? • How do I interact with my environments? • What environments are part of my immediate world?	**Awareness that environments are affected by people, organizations and other environments.** Students will demonstrate this by: • Describing the impacts, both positive and negative, of organizations on environments. • Giving examples of how one environment can benefit or harm another. • Showing awareness of people's ability to both create and destroy environments. • Identifying organizations that impact various environments.	• How do various environments affect people? • What are ways organizations impact environments? • How do various different environments interact?
Reflection on:	Grades 7 and 8		Grades 9 and 10	
Responsibilities	**How our actions can benefit, harm or change environments.** Students will demonstrate this by: • Indicating specific examples of the negative impact they have on their environments. • Indicating specific examples of the positive impact they have on their environments.	• How can we make informed and responsible choices? • What are the lifestyle implications of making environmental choices?	**How collective actions can benefit, harm or change environments.** Students will demonstrate this by: • Identifying established organizations that promote change within environments. • Illustrating with specific examples ways that organizations have harmed or helped environments.	• What affiliations do I have with organizations that promote change within environments? • What are the differences between collective and individual action?
Action on:	Grades 7 and 8		Grades 9 and 10	
Issues	**Take individual action on issues in your immediate environments; school and home.** Students will demonstrate this by: • Outlining individual ways they can help contribute to solving an environmental problem. • Actively promoting change in their immediate environments.	• What difference can I make as an individual?	**Create, join or support organizations or endeavors that are working towards positive changes in larger scale environments.** Students will demonstrate this by: • Contributing to organized efforts to improve environments. • Evaluating the validity and effectiveness of various different endeavors in which they participated.	• How can I be involved on larger scale?

Student Learning Outcome Areas	Student Expectations	Key Questions	Student Expectations	Key Questions
Awareness and understanding of:	Grades 7 and 8		Grades 9 and 10	
Community	*Understand what a community is. Awareness of the roles they play in their community. Understanding the importance of the different roles and relationships in a community.* Students will demonstrate this by: • Describing their community and elements of it that make it unique. • Recognizing their current role in their community and how they contribute. • Explaining various ways that members of their community can both change and contribute to the community.	• How does the material we our studying help me to identify my community? • How does this unit help me understand my place in my community? • How does the topic we are discussing both affect and alter my community and its members?	*Understand the dynamics of the various communities within a larger community and which different communities they are members of. Understanding of how the various communities impact their lives.* Students will demonstrate this by: • Listing and defining several unique community groups that they are involved in. • Identifying and understanding the ways in which various community groups both identify themselves and integrate themselves within larger communities. • Recognizing of the impacts of other community groups on their lives.	• How does the material we are covering illustrate the existence of community groups? • How might different community groups impact, be affected by and relate to the material we are studying? • How does this unit we are studying help me better understand my community as a whole?
Reflection on:	Grades 7 and 8		Grades 9 and 10	
Attitudes	*Discovering the causes and patterns of different social perspectives and reflecting on the impact they have on the community and on social harmony.* Students will demonstrate this by: • Explaining the community wide implications of their actions. • Describing the need for a variety of perspectives in a community.	• How do my choices affect my community? • What benefits are there to understanding other points of view?	*Realization of the various factors and attitudes that contribute jointly to social discord and that only through understanding and cooperation can solutions be found.* Students will demonstrate this by: • Illustrating why communities need help from others. • Explaining the benefits that can be obtained through community action.	• How can my learning help me identify areas of need in various communities?
Responsibilities	*Discovering the foundations of meaningful community service through uncovering the ethical implications of activity or inactivity.* Students will demonstrate this by: • Describing the different ways that we can help. • Outlining their duties and responsibilities as an active contributor to their community.	• How have we responded to areas of need? • What should I do?	*Through meaningful reflection on social situations and areas of need, realizing where we can be an active contributor and in which areas we can continue to develop empathy.* Students will demonstrate this by: • Using their understanding of social need to create and develop solutions. • Showing mature understanding of and compassion for global concerns.	• How can I be of service?
Action:	Grades 7 and 8		Grades 9 and 10	
Involvement through service	*Recognizing the various types, levels and areas of community involvement available to them.* Students will demonstrate this by: • Participating in a variety of guided service endeavors. • Differentiating between, fundraising, active service and community involvement.	• What needs to be done? • What can I do?	*Continuously striving to be an active and contributing member of our community through our willingness to become independently engaged in helping others and bettering our communities.* Students will demonstrate this by: • Actively seeking out and being a participant in varied community service endeavors independently. • Being a role model for community service and involvement	• What **will** I do? • How do I encourage others to help? • What difference do my contributions make? • How is my involvement contributing to my personal growth?

ATL skill area	Student Learning Expectations Demonstrated by:	Key questions for use with MYP units of work	Student Learning Expectations Demonstrated by:	Key questions for use with MYP units of work
	Grades 7 and 8		Grades 9 and 10	
Organization	**Task management** • outlining steps for completing a task **Self-management** • organizing learning materials, i.e. research sources, texts, digital resources, etc., for use in completing tasks **Time management** • using time effectively in class; avoiding procrastination	• What organizational tools do I have? • Why should I organize my time?	**Task management** • Independently analysing and prioritizing the steps necessary to most efficiently complete a task **Self-management** • Effectively prioritizing and balancing personal interests and school commitments • Reflecting on the correlation between organizational skills and success **Time management** • Scheduling time efficiently and developing methods to meet all commitments	• How can I best organize myself? • What aspects of my organization do I need to develop? • How do I best organize tasks?
Collaboration	**Working in groups** • Taking on various roles within a group • Demonstrating teamwork **Accepting others** • Listening to and being respectful of all ideas **Personal challenges** • Being respectful of differences	• What roles exist in a group? • What is the value of working together?	**Working in groups** • Assigning roles and accepting the roles assigned to you • Acting as a responsible member of a team **Accepting others** • Being open to and providing constructive feedback **Personal challenges** • negotiating goals and limitations with peers and teachers	• How can I be a responsible group member? • How can I improve the dynamics of a group?
Communication	**Literacy** • Using and developing appropriate reading strategies for garnering information from print and digital media **Presentation Skills** • Finding effective and appropriate methods for relaying information **Interpersonal Skills** • Choosing appropriate tone and method for communicating a variety of audiences	• Why do I need to be an effective communicator? • How do I effectively demonstrate my understanding?	**Literacy** • Developing and using effective reading and writing skills to meet the needs of all subject areas • Mastering a wide range of subject specific terminology **Presentation Skills** • Using new and dynamic ways to present information in an engaging manner **Interpersonal Skills** • Being an effective, responsible communicator who is aware of the various differences in the ways we communicate	• How do I effectively help others understand? • How do the ways in which I communicate with others reflect on me?

Information Literacy	**Accessing Information** • Identifying primary and secondary sources • Choosing relevant and appropriate sources **Selecting and Organizing Information** • Identifying points of view and bias • Recognizing the variety of media that influences their lives **Referencing** • Creating properly formatted MLA style works cited lists • Using inline referencing for direct quotations • Understanding all the types of sources they must cite and respecting the concept of intellectual property rights	• How do I know if the information is reliable? • How do I acknowledge the ideas of others?	**Accessing Information** • Using sources to support original ideas • Using a variety of academic resources **Selecting and Organizing Information** • Understanding the foundations of bias • Recognizing the pervasive impact of media in their lives **Referencing** • Using inline referencing for all quotations , paraphrases and summarizations	• How can information help me enhance my ideas? • Why are we biased?
Reflection	**Self-awareness** • Seeking feedback and understanding on how to improve **Measuring** • Keeping portfolios and reflecting at regular intervals and different stages in the learning process	• How do I reflect? • How have my reflections helped me learn?	**Self-awareness** • Identifying personal strengths and weaknesses. • Overcoming limitations and maximizing potential. **Self-evaluation** • Analysing and recording achievements and progress	• What do I do with my reflections? • How do I continue to improve?
Thinking	**Investigating and Planning** • Establishing ideas and planning through the use of graphic organizers, brainstorming, discussion, outlining and prewriting strategies • Inquiring into topics, gathering data, identifying perspectives, creating questions and looking for solutions using the inquiry cycle **Application** • Using knowledge to create logical order arguments • Identifying problems and the attempts to resolve them then developing creative	• How do I manage my thoughts? • What tools can help me think in different ways?	**Investigating and Planning** • Creating a plan for gathering information and finding solutions • Challenging information and existing perspectives using the inquiry cycle **Application** • Identifying patterns in information • Critically selecting the most relevant information to create insightful and persuasive arguments • Considering multiple perspectives on a issue, analysing the effectiveness of solutions and identifying the most	• How do I find the best outcomes? • How can I develop an effective plan?

Transfer	**Making Connections** • Using common skills to solve problems across subject areas • Identifying common themes and concepts **Real World Application** • Understanding in what ways their learning impacts their lives.	• What are the "big ideas" of each of the different subjects? • Do the big ideas of the subjects overlap?	**Making Connections** • Identifying common skills • Consolidating skills to handle unfamiliar situations **Real World Application** • Using their knowledge and skills to make positive changes and create effective solutions to better their lives.	• How can I use my knowledge, understanding and skills across subjects? • How does my learning improve my life?

Learner-Centered Instruction
By: Paul Kim

Model	Attributes
Inquiry	• A learner-centered, active learning approach focusing on questioning, critical thinking, and problem solving • Follows the principle that involving learners will help them better understand the lessons
Resource-based learning	• Learners actively engage in multiple resources (print and non-print) • Learners responsible for selecting resources (e.g. Internet, books, human) that appeal to their personal learning preferences, interests and abilities • Learners become active learners as they use a wide range of materials to investigate subject material prescribed within their classroom curriculum
Cognitive Apprenticeship	• Learners work in teams on projects or problems with close scaffolding of the teacher • Guided participation helps the learner achieve tasks that independently would be too hard or complicated. • The task or goal is to form a process of thinking—or something that is intangible into something tangible • Teachers usually model or scaffold the skills or tasks in the beginning. Once learners begin to understand, the modeling and scaffolding is reduced. This allows learners to accomplish the task on their own and only ask for help when needed
Problem-based learning	• Focuses on the process of problem solving, critical thinking in situated contexts, and acquiring knowledge. It is inquiry-based when learners are active in creating the problem • Emphasis is placed on using communication, cooperation, and resources to formulate ideas and develop reasoning skills • Knowledge is constructed within each individual or community based on the learner's or community's prior knowledge, values, beliefs, and perspectives. • Learning occurs through social interactions whereby an outside source can help individuals extend their learning • Activities are organized around achieving a shared goal (project)
Project-Based Learning	• Focuses on developing a product or creation • Engages learners by starting with concrete and solving hands-on, real-world problems • Learners are usually provided with specifications for a desired end product (e.g a specific project, such as building a rocket or designing a web site) • The learning process is more oriented to following correct procedures. • Teachers are more likely provide expert guidance, feedback and suggestions (e.g. modeling, scaffolding) to help learners achieve the final product. This is provided according to learner needs and within the context of the project • Activities are organized around achieving a shared goal (project)
Collaborative Learning	• Learners placed in groups or pairs for the purpose of achieving a common academic goal • Learners are responsible for one another's learning as well as their own. Thus, the success of one learner helps other learners to be successful • Does not require face-to-face interaction as collaborative learning can take place across the Internet
Cooperative Learning	• It is a specific kind of collaborative learning, where learners work together in *small* groups on a structured activity. They are individually accountable for their work and are responsible for helping teammates learn • Cooperative groups work face-to-face and learn to work as a team
Constructivism	• Founded on the premise that reflecting on personal experiences allows learners to construct their own understanding of the world • Teachers focus on making connections between facts and fostering new understanding in learners • Teachers rely heavily on open-ended questions and promoting extensive dialogue among learners • Learners encouraged to analyze, interpret, and predict information

Interdisciplinary projects

Simultaneous projects	Sequential projects
• The team of teachers chooses a broad concept. • Subject groups that can contribute to the concept join.	• Project achieved in stages. • Concepts from subject groups needed in order to go further.
Theme X → Subject A → Subject B → Subject C → Etc.	**A → B → C → Objective**

What is a unit question ?

A unit question is a multifaceted broad-based question that frames a specific unit. It should help students to focus on the essence of the unit.

Unit questions

should **not** be content-centered; but they may be…

- **abstract**
- **realizations**
- **raising awareness**
- **concepts**
- **perspectives**
- **real world applications**

Unit questions

- should hook students and give them a reason to care about the subject
- should integrate an area of interaction
- should be student-friendly

Is it a good unit question?

- Is the question open-ended? **YES!**
- Is the question relevant and engaging? **YES!**
- Is the question challenging and provocative? **YES!**
- Is the question significant? **YES!**
- Can the student find the answer to this question in a book? **NO!**

Stage 1 – Integrate significant concepts, areas of interaction and unit question

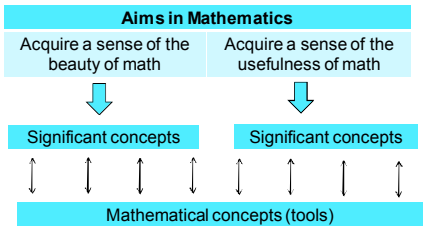
Area of interaction focus	Significant concept(s)
Which area of interaction will be our focus? Why have we chosen this?	What are the big ideas? What do we want our students to retain for years into the future?

MYP unit question

International Baccalaureate

Significant concepts in MYP Mathematics

What are significant concepts?



Acquire a sense of the beauty of math

- System: cleanliness, excitement, fear, it all fits together (serendipity)
- Aesthetics of math in nature (symmetry in faces)
- Simplicity to understand complexity
- Order in chaos
- There is an answer (ugly and elegant answers)

Acquire a sense of the usefulness of math

- Math in daily life
- Using the abstract to understand the concrete
- Tools for other disciplines (architecture, science.....)
- Measurement
- Prediction
- Processing information / chunking information
- Explaining relationships
- Decision making /persuasion
- Spatial orientation (navigation, finding the way, etc)

Mathematical concepts (tools)

- Language of mathematics
  - Symbols
  - Graphs
  - Tables
  - 2D and 3D representations
  - Formulas
- Analytical thinking
  - Logic: Proofs, deduction and induction, discerning truth
  - Reflection
  - Justification
  - Determining accuracy
- Number

Stage 1 – Integrate significant concepts, areas of interaction and unit question

Area of interaction focus	Significant concept(s)
Which area of interaction will be our focus? Why have we chosen this?	What are the big ideas? What do we want our students to retain for years into the future?
Community and service: the drop-off and pick-up from school as a chaotic situation which might be improved on.	Order in chaos: appreciation of the power of mathematics to find order in chaos

MYP unit question

How can we improve the traffic flow in and around our school at the beginning and end of school days?

Assessment
What task(s) will allow students the opportunity to respond to the unit question? What will constitute acceptable evidence of understanding? How will students show what they have understood?
-Make an inventory of the types of transportation used (students will gather data and represent appropriately) -Investigate the journeys and movement that take place (students will represent journeys in mathematical form: vectors, networks, diagrams, etc) -Parking situation, drop off situation (students will make a mathematical model of the traffic flow) -Make an improvement plan to present to school management
Which specific MYP objectives will be addressed during this unit?
-Knowledge and understanding: methods of gathering data, understanding and using networks and vectors to understand real life situations -Investigating patterns: modelling -Communication -Reflection: how accurate, how realistic is model (can chaos be modelled?); what is appropriate action
Which MYP assessment criteria will be used?
all

Stage 2 – Backward planning: from the assessment to the learning activities through inquiry

Content
What knowledge and/or skills (from the course overview) are going to be used to enable the student to respond to the unit question? What (if any) state, provincial, district, or local standards/skills are to be addressed? How can they be unpacked to develop the significant concept(s) for stage 1?
-mathematical modelling **-gathering data** **-networks (new for students)** **-vectors (new for students)**

Objectives for years 1, 3 and 5 of the Middle Years Programme

Year 5 objectives

The mathematics objectives for year 5 of the Middle Years Programme (MYP) are already in place and can be found in this guide. This set of **prescribed** objectives forms the basis for the **assessment criteria**, also published in the guide, which must be used for final assessment of students' work during year 5.

Example interim objectives

Example interim objectives for years 1 and 3 of the MYP appear in the tables that follow. They have been developed in order to:

- promote articulation between the MYP and the Primary Years Programme (PYP)
- support individual schools in developing a coherent curriculum across the five years of the programme (or however many years a school is authorized to offer)
- emphasize the need to introduce students to the required knowledge, understanding, skills and attitudes from the first year of the programme
- provide examples of possible learning experiences that will allow students to work towards meeting the final objectives for year 5
- support schools that are authorized to offer the first three years of the MYP in designing appropriate assessment tasks for the end of the third year.

Unlike the objectives for year 5, the interim objectives for years 1 and 3 are not prescribed**,** although the IB recommends that all schools use them. Schools may choose to adopt the objectives contained in this document or develop their own.

If choosing to develop their own interim objectives, schools must start with the prescribed objectives for year 5 and modify each one by taking into account the age, prior knowledge and stage of development of students in an earlier year of the programme. Each year 5 objective will then correspond directly to a modified objective in a preceding year of the programme. **No objectives should be omitted** from a previous year as it is vital to ensure a coherent progression of learning across all five years of the programme.

MYP units of work

Examples of possible learning experiences, each aligned to an objective, appear in the tables that follow. Each learning experience is intended to form part of a larger unit of work designed to address a central question or theme, known as the **MYP unit question**. More information about MYP units of work can be found in the section on "Planning for teaching and learning" in *MYP: From principles into practice* (August 2008).

Within each unit of work, the **context for learning**, **significant concept(s)** and **assessment tasks** are defined in relation to the MYP unit question. The areas of interaction provide the context for learning while the significant concepts refer to the underlying concepts that define the principal goal of the unit. Assessment tasks are designed to address the levels of students' engagement with the MYP unit question and the aligned objectives.

Context for learning

Every MYP unit of work has an approaches to learning (ATL) component: a shared and agreed set of skills that all teachers develop with their students throughout the entire programme. The context that frames a particular unit of work is generally derived from one of the other four areas of interaction, although ATL might be the specific context on some occasions. Some of the examples of learning experiences listed in the tables that follow have an obvious connection to one of the areas of interaction, for example, investigating the relationship between the volume of air in a classroom (and other enclosed spaces) and the health requirements of each student. Other connections may become clear only after a more considered approach but teachers should be able to establish these connections for their own students within each MYP unit of work.

Several examples of learning experiences listed also strongly suggest the possibility of planning an interdisciplinary unit in collaboration with other subject teachers, for example, representing Newton's laws of motion as algebraic equations, tables and graphs using data that has been generated and data that has been collected experimentally.

Assessment tasks

One of the first stages in planning a unit of work is to design **summative assessment tasks**, linked to the MYP unit question, which provide varied opportunities for students to demonstrate their knowledge, understanding, skills and attitudes. It is also important to include ongoing **formative assessment tasks** within a unit of work as these provide valuable insights into the extent of student learning as the unit of work progresses.

It is important to realize that the formats of both summative and formative assessment tasks need not be reduced to examinations, tests, quizzes and written questions set as homework. These formats are valid in certain cases but do not always take into account different learning styles and may not provide students with sufficient creative scope to demonstrate all they have learned. There are many different ways in which evidence of student learning can be found. For example, students could carry out assessment tasks that involve:

- making a presentation using visual aids (for example, flipcharts, electronic slides)
- solving a cross-number puzzle where the clues are provided in the form of calculations to be made and/or problems to be solved
- playing a game that requires a particular set of skills or knowledge and understanding of certain concepts
- making a three-dimensional model (for example, scale models of the earth, moon and sun)
- telling a story (for example, stories where numbers have been deliberately scaled up or down by multiples of ten to provide comic entertainment, thereby demonstrating the need for accuracy with regard to place value)
- keeping a personal journal that documents their development of mathematical understanding
- making a poster or wall chart
- writing a short song or poem that incorporates important mathematical principles (for example, a rap chant incorporating the principles of Pythagoras' theorem)
- creating a mnemonic as an aid to memory (for example, the rules for the sine, cosine and tangent properties of a right-angled triangle expressed in one word, SOHCAHTOA, or as a phrase or saying, "**S**ome **o**wls **h**ave ….")
- keeping a scrapbook containing extracts from the media that illustrate a particular property
- maintaining a folder of their own work
- developing an information booklet/leaflet that describes a concept and/or mathematical process in detail
- writing a summary sheet as a revision guide for a particular mathematical topic
- creating pictures, diagrams or cartoons to illustrate a particular concept or process
- carrying out an investigation
- collecting data and storing it in appropriate formats (for example, tables, spreadsheets)
- creating a personal data booklet.

Tables of objectives

A Knowledge and understanding

Knowledge and understanding are fundamental to studying mathematics and form the base from which to explore concepts and develop problem-solving skills. Through knowledge and understanding students develop mathematical reasoning to make deductions and solve problems.

Year 1	Year 3	Year 5
Objectives		
At the end of the first year, students should be able to demonstrate basic knowledge and understanding of the following branches of mathematics: • number • algebra • geometry and trigonometry • statistics and probability • discrete mathematics by being able to:	At the end of the third year, students should be able to demonstrate some knowledge and understanding of the following five branches of mathematics: • number • algebra • geometry and trigonometry • statistics and probability • discrete mathematics by being able to:	At the end of the course, students should be able to demonstrate knowledge and understanding of the following five branches of mathematics: • number • algebra • geometry and trigonometry • statistics and probability • discrete mathematics by being able to:
• know and demonstrate understanding of some of the basic concepts of number, algebra, geometry and trigonometry, statistics and probability, and discrete mathematics	• know and demonstrate understanding of some of the concepts of number, algebra, geometry and trigonometry, statistics and probability, and discrete mathematics	• know and demonstrate understanding of the concepts from the five branches of mathematics (number, algebra, geometry and trigonometry, statistics and probability, and discrete mathematics)
• use basic concept-specific strategies to solve simple problems in both familiar and unfamiliar situations including those in real-life contexts	• use appropriate mathematical concepts and skills to solve simple problems in both familiar and unfamiliar situations including those in real-life contexts	• use appropriate mathematical concepts and skills to solve problems in both familiar and unfamiliar situations including those in real-life contexts
• apply basic rules correctly to solve simple problems including those in real-life contexts.	• select and apply basic rules correctly to solve problems including those in real-life contexts.	• select and apply general rules correctly to solve problems including those in real-life contexts.
Examples of possible learning experiences		
Number Students could: • find data in newspapers and classify it as discrete or continuous • classify numbers as natural, odd, even, square and/or triangular • use the Sieve of Eratosthenes to find prime numbers less than 100	**Number** Students could: • make a geological time line on the wall of the classroom • investigate the relationship between the volume of air in a classroom (and other enclosed spaces) and the health requirements of each student	**Number** Students could: • compare the number of kilometres per litre of fuel per passenger used by planes, trains, buses and/or cars • explore the history and significance of irrational numbers and identify some of the symbols used for particular irrational numbers, for example, π (pi), e (Euler's number), φ (the golden ratio).

- count very large sets of objects (coins, cars passing, people in a large space, numbers) to emphasize the importance of organization and grouping
- make a time line on the wall of the classroom emphasizing dates of important mathematicians and mathematical discoveries.

Algebra

Students could:

- use a pan balance to simulate the addition or subtraction of like quantities from both sides of an equation by keeping the pans balanced
- use the balance model to create equations for classmates to solve
- create pictures on squared paper and provide a list of coordinates for classmates to construct the picture by joining the points in order
- create a booklet containing information on algebra topics (explanations, examples and exercises) for use by subsequent students.

Geometry and trigonometry

Students could:

- investigate the sum of the interior angles of triangles and quadrilaterals by drawing and cutting out different shapes on paper, tearing off the angles and fitting them together to form straight lines and/or circles
- investigate the tangent ratio by comparing students' heights with the lengths of their shadows
- investigate reflections by using a mirror to reflect faces along different centre lines.

Statistics and probability

Students could:

- find the ranges and means of leaf lengths on two trees of the same species but located in different environments (sun and shade).
- investigate rounding numbers to a specified number of significant figures by considering the accuracy of measurements in real life, such as the length of a 100 m athletics track, the extent of possible errors and the impact these may have.

Algebra

Students could:

- generate a series of ordered pairs by substituting values in a linear equation and have classmates identify patterns and/or work out the formula
- find a rule for the number of 1 cm squares needed to put a 1 cm-wide frame around a square picture whose side is n cm.

Geometry and trigonometry

Students could:

- draw or construct a model of an appropriate building (the school or their own house) by piecing together rectangular or triangular prisms only
- estimate the volumes of irregular solid objects as the sum of more simple approximated shapes and verify their results by immersing each object in water and measuring the displaced volume
- investigate the underlying patterns and constructions evident in particular designs and/or artworks, for example, traditional Moroccan designs, and then create their own tessellated designs
- use Pythagoras' theorem and the trigonometric ratios as tools for measuring large-scale objects and/or distances in open spaces.

Statistics and probability

Students could:

- design statistical surveys to investigate health and social education issues, with guidance from the teacher
- construct a tree diagram for a three-day weather forecast where the probability of rain on any day is estimated from past data
- compare the lengths of words and/or sentences in texts aimed at different readerships.

Algebra

Students could:

- show that the ratios of successive terms of a Fibonacci sequence (u_n/u_{n+1}) converge to the same value regardless of the term chosen as u_1
- find the best angle for throwing a basketball so that it will go in the basket from the free throw line, by modelling its trajectory graphically using their knowledge of quadratic equations
- investigate exponential growth in a biological population.

Geometry and trigonometry

Students could:

- use the unit circle as a physical tool to calculate the values of different trigonometric ratios (in order to appreciate the circular nature and symmetry of each function, and the significance of the asymptotes in the tangent function)
- use the transformations of translation, reflection, rotation, enlargement and shear to describe the actions of a particular cartoon character
- carry out research into the history of angle measurement and the introduction of trigonometry.

Statistics and probability

Students could:

- collect information relating to used cars for sale (mileage, age, make, engine size, cost now, cost when new) and explore the relationships between different pairs of variables
- select several countries and look for key statistics (population growth, average income and life expectancy) on the Internet in order to answer questions such as, "Do people in richer countries appear to live longer?", "Is there any relation between the size of a population and its average income?"

Discrete mathematics Students could: • draw two large intersecting circles on the floor and identify two categories of student, for example, {girls} and {students wearing something blue}, and move into one of the four defined regions according to their characteristics • play mathematical games (for example, “bingo”, by calculating the answers to simple problems read out by the teacher) to review and reinforce previous learning.	**Discrete mathematics** Students could: • create a minimum network for broadband cables to connect five major cities in their country • create a Koch snowflake by starting with a large equilateral triangle, dividing each side into three equal parts, removing the middle part and replacing it with two sides of a triangle equal in length to the part that was removed, and so on.	**Discrete mathematics** Students could: • solve a logical puzzle, such as the following: “You have three boxes of fruit, one with apples, one with oranges and one mixed; each box is labelled but the labels are not on the correct boxes of fruit—how can you know what each box contains simply by taking one piece of fruit from one box?” • create their own logic puzzle by using websites such as http://www.edhelper.com/logic_puzzles.htm) • play the chaos game (refer to http://home.inreach.com/kfarrell/fractals.html).

B Investigating patterns

Investigating patterns allows students to experience the excitement and satisfaction of mathematical discovery. Mathematical inquiry encourages students to become risk-takers, inquirers and critical thinkers. The ability to inquire is invaluable in the MYP and contributes to lifelong learning.

Through the use of mathematical investigations, students are given the opportunity to apply mathematical knowledge and problem-solving techniques to investigate a problem, generate and/or analyse information, find relationships and patterns, describe these mathematically as general rules, and justify or prove them.

Year 1	Year 3	Year 5
Objectives		
At the end of the first year, when investigating problems, in both theoretical and real-life contexts, students should be able to:	At the end of the third year, when investigating problems, in both theoretical and real-life contexts, students should be able to:	At the end of the course, when investigating problems, in both theoretical and real-life contexts, students should be able to:
• apply basic inquiry and mathematical problem-solving techniques, with guidance from the teacher, by identifying variables, posing relevant questions, organizing data and using an appropriate model	• select and apply basic inquiry and mathematical problem-solving techniques to problems by asking searching questions	• select and apply appropriate inquiry and mathematical problem-solving techniques
• recognize simple patterns similar to previously seen examples	• recognize simple patterns in different situations	• recognize patterns
• describe simple patterns in words and/or diagrams	• describe simple patterns as relationships or general rules	• describe patterns as relationships or general rules
• arrive at a result or set of results and make predictions based on extending the pattern(s)	• arrive at a single result or set of results and make predictions consistent with findings	• draw conclusions consistent with findings
• describe simple mathematical relationships.	• explain simple mathematical relationships and general rules using logical arguments.	• justify or prove mathematical relationships and general rules.
Examples of possible learning experiences		
Number Students could: • predict the next three numbers in a sequence • find large numbers that can be reduced to prime factors by their classmates • investigate how many different designs can be made by shading squares in a 3 x 3 square. **Algebra** Students could: • investigate the graphs of $y = mx + c$ using appropriate software • measure and plot the extended length of a spring against the weight of an object hung on the end	**Number** Students could: • determine ways of finding the sum of the terms in an arithmetic sequence, describing their methods in general terms • investigate the meaning of negative exponents using a calculator • investigate the patterns present in the Fibonacci sequence. **Algebra** Students could: • investigate the graphs of $y = (x - a)(x - b)$ and their solutions for different values of y using a graphic display calculator.	**Number** Students could: • be given the diameter of a model of the earth (a globe) as 100 cm, and investigate the diameters of corresponding models of the planets, given their true diameters expressed in standard form ($a \times 10^n$), and discuss the status of Pluto as a planet. **Algebra** Students could: • analyse and compare male and female record times in a particular sport (running, swimming) from 1900 to the present day, and predict times for the year 2050 by investigating lines or curves of best fit for the data collected.

• investigate problems such as the following: "How long does it take to make *n* pieces of toast on a one-sided grill that can take two pieces at a time?" **Geometry and trigonometry** Students could: • investigate the sum of the interior angles of different *n*-sided polygons ($n > 2$) • use different instruments to measure a range of objects and discuss the ease and accuracy of each technique. **Statistics and probability** Students could: • investigate the probabilities for the different outcomes when tossing two coins or rolling two dice. **Discrete mathematics** Students could: • use Venn diagrams to analyse different aspects of after-school activities.	**Geometry and trigonometry** Students could: • write a logical explanation for the sum of the exterior angles of a polygon equalling 360^{o} • investigate how to measure tall structures, telegraph poles, buildings • use geometry to predict the angle of the noontime sun at the two solstices. **Statistics and probability** Students could: • choose, with logical explanations, which measure of central tendency would be most appropriate for typical family size, height of students, amount of pocket money**.** **Discrete mathematics** Students could: • investigate shortest paths in local networks (bus routes, metro lines) • investigate the concept of map colouring by conducting research into the four-colour problem and attempting to illustrate this rule by colouring a map of their country showing the different regions/states.	**Geometry and trigonometry** Students could: • carry out a survey of the school grounds for the purpose of creating a detailed and accurately scaled map/plan • investigate how a sphere can be projected onto a plane. **Statistics and probability** Students could: • design a questionnaire to elicit data reflecting attitudes to issues relevant to them; circulate this questionnaire to students in their own school and one other school, possibly in another country; then collect and compare the data • create a fund-raising game of chance that is profitable and will attract players. **Discrete mathematics** Students could: • investigate whether, when you start with an odd number *a*, square it to give *b*, subtract 1 to give *c*, divide by 2 to give *d*, and add 1 to give *e*, the equation $a^2 + d^2 = e^2$ will always be satisfied.

C Communication in mathematics

Mathematics provides a powerful and universal language. Students are expected to use mathematical language appropriately when communicating mathematical ideas, reasoning and findings—both orally and in writing.

Year 1	Year 3	Year 5
Objectives		
At the end of the first year, students should be able to communicate mathematical ideas, reasoning and findings by being able to:	At the end of the third year, students should be able to communicate mathematical ideas, reasoning and findings by being able to:	At the end of the course, students should be able to communicate mathematical ideas, reasoning and findings by being able to:
• use appropriate mathematical language (notation, symbols, terminology) in both oral and written communications, with guidance from the teacher	• use appropriate mathematical language (notation, symbols, terminology) in both oral and written explanations in familiar situations	• use appropriate mathematical language (notation, symbols, terminology) in both oral and written explanations
• use different forms of mathematical representation (simple formulae, diagrams, tables, charts, graphs and models), with guidance from the teacher	• use different forms of mathematical representation (simple formulae, diagrams, tables, charts, graphs and models)	• use different forms of mathematical representation (formulae, diagrams, tables, charts, graphs and models)
• state, in writing and/or verbally, the steps followed in solving simple problems.	• communicate a mathematical line of reasoning in solving simple problems using different forms of representation.	• communicate a complete and coherent mathematical line of reasoning using different forms of representation when investigating complex problems.
Examples of possible learning experiences		
Number Students could: • modify quantities and measurements in well-known events or stories to create absurdities and then act out or model the event or story, for example, speed limits being set at 50 m per hour (instead of 50 km per hour); playing on a football pitch where it is assumed that 1 m^2 = 100 cm^2 • investigate discounts in advertisements in order to determine the best deals. **Algebra** Students could: • explain the steps involved in solving a linear equation • explain the significance of m and c when the line represented by $y = mx + c$ is graphed.	**Number** Students could: • collect media clippings of the inappropriate use of mathematical symbols and terminology, and/or inappropriate representations of data (for example, misleading labelling on graphs) and describe how these could lead to incorrect interpretations. **Algebra** Students could: • use motion recording equipment to create distance/time graphs • create feasible distance/time graphs and physically model the graphs created by others by moving around the classroom.	**General** Students could: • take it in turns to summarize the important elements of selected lessons by putting the information into a class file or posting it on a website in the form of a blog; the document could then be used as a revision tool by the class. **Number** Students could: • be given a series of real-life calculations to carry out using a calculator (for example, "A third of the people in a city with a population of 500,000 live in poverty—how many people live in poverty?") and be asked to justify the number of significant figures given in their answers.

Geometry and trigonometry Students could: • make a scale drawing of a bicycle • investigate the properties of similar two-dimensional shapes (triangles, squares and circles). **Statistics and probability** Students could: • measure the lengths of their ears and represent the data in tables and graphs, and find measures of central tendency • investigate the probability of various events taking place based on available data (for example, the probability of rain on a particular day of the year). **Discrete mathematics** Students could: • use Venn diagrams to classify quadrilaterals that have equal sides, parallel lines, equal angles • draw diagrams showing three different routes they could take to travel from home to a specific destination and determine the best one by considering either times or distances.	**Geometry and trigonometry** Students could: • write out directions, using bearings and distances, to describe the cycle routes in their local area • carry out a survey of their school grounds/campus in order to create an accurate plan. **Statistics and probability** Students could: • collect data from a weather station and represent it graphically as a tool for investigating trends • discuss the advantages and disadvantages of obtaining data by sampling large populations, by referring to sampling techniques used by the media (for example, opinion polls). **Discrete mathematics** Students could: • investigate the similarities and differences between the skills needed for two different occupations using Venn diagrams (for example, music teacher and rock star) • create flow charts to describe some simple mathematical processes (for example, finding the greatest common divisor of two numbers).	**Algebra** Students could: • be given an open-ended problem with fixed and variable costs, such as budgeting for an event, where the solution depends on the parameters, expected outcomes and accuracy of estimates • investigate Newton's laws of motion in the form of algebraic equations, tables and graphs using generated data and data that has been collected experimentally • investigate and describe the trajectory of small objects falling from different forms of transport (bicycles, lorries/trains, hot-air balloons). **Geometry and trigonometry** Students could: • design an orienteering course on the sports field where the beginning and end points coincide, and use practical trials and/or the sine and cosine rules to demonstrate that these points are coincident. **Statistics and probability** Students could: • investigate different methods of sampling large populations (random sampling, stratified sampling, systematic sampling, cluster sampling, convenience sampling) and create a poster explaining each one. **Discrete mathematics** Students could: • conduct a poll among themselves to determine their preferences for different types of music; collate the results in the form of a Venn diagram; then describe the information displayed.
Students are encouraged to choose and use information and communication technology (ICT) tools as appropriate and, where available, to enhance communication of their mathematical ideas. ICT tools can include calculators (simple, scientific and/or graphic display), screenshots, graphing, spreadsheets, databases, and drawing and word-processing software.		

D Reflection in mathematics

MYP mathematics encourages students to reflect upon their findings and problem-solving processes. Students are encouraged to share their thinking with teachers and peers and to examine different problem-solving strategies. Critical reflection in mathematics helps students gain insight into their strengths and weaknesses as learners and to appreciate the value of errors as powerful motivators to enhance learning and understanding.

Year 1	Year 3	Year 5
Objectives		
At the end of the first year, students should be able to:	At the end of the third year, students should be able to:	At the end of the course, students should be able to:
• consider the reasonableness of their results in the context of the problem	• consider the reasonableness of their results in the context of the problem and attempt to explain whether they make sense	• explain whether their results make sense in the context of the problem
• consider the importance of their findings, with guidance from the teacher	• consider the importance of their findings	• explain the importance of their findings
• distinguish between measurement and counting, and demonstrate an appreciation of the difference between degrees of error in measuring and mistakes in counting, measuring and calculating	• consider the degree of accuracy of their results where appropriate and estimate errors in simple measurements	• justify the degree of accuracy of the results where appropriate
• consider alternatives to the method when appropriate, with guidance from the teacher.	• consider alternatives to the method when appropriate.	• suggest improvements to the method when necessary.
Examples of possible learning experiences		
Number Students could: • estimate answers before carrying out calculations • discuss the precision of different measuring instruments (rulers, callipers, protractors, theodolites). **Algebra** Students could: • substitute answers in original problems to check results. **Geometry and trigonometry** Students could: • find the least and greatest amount of paint necessary to paint their bedrooms.	**Number** Students could: • investigate how well the "golden ratio" applies to famous buildings or paintings. **Algebra** Students could: • determine the domain and/or range of functions involving physical processes, for example, the elastic limit of a rubber band as a linear function within a limited domain. **Geometry and trigonometry** Students could: • design an economical shape for a 500 ml bottle of soda • use Pick's theorem to estimate the area of regions on a map	**Number** Students could: • discuss the appropriateness of degrees of accuracy for different data/information found in the media. **Algebra** Students could: • discuss when substitution, graphing or elimination are the most appropriate strategies for solving different sets of simultaneous equations • investigate whether a table-tennis ball rolled off a desk will follow a parabolic path. **Geometry and trigonometry** Students could: • make a sundial and investigate the position of the shadow at different times of the year compared to the pre-calculated positions.

Statistics and probability Students could: • draw a bar chart showing the hours of television watched each day by classmates • collect examples from the media of data displayed in different ways (bar graphs, pie charts, pictograms) and comment on their effectiveness. **Discrete mathematics** Students could: • invent a problem similar to the “Bridges of Königsberg”.	• scale up the measurements of a popular doll or toy model to determine whether their measurements lead to absurdities when their proportions are scaled to a realistic level. **Statistics and probability** Students could: • analyse the data collected to investigate an environmental problem. **Discrete mathematics** Students could: • create and model a traffic problem involving one-way and two-way streets.	**Statistics and probability** Students could: • investigate, by collecting appropriate data, whether the annual harvest of a particular fish species justifies the creation of conservation laws. **Discrete mathematics** Students could: • devise emergency exit paths to support the swift and safe evacuation of students and staff in their school.

Presumed knowledge for mathematics SL and HL

General

Students are not required to be familiar with all the topics listed as presumed knowledge (PK) **before** they start this course. However, they should be familiar with these topics before they take the **examinations**, because questions assume knowledge of them.

Teachers must therefore ensure that any topics designated as presumed knowledge that are unknown to their students at the start of the course are included at an early stage. They should also take into account the existing mathematical knowledge of their students to design an appropriate course of study for mathematics SL or mathematics HL.

This list of topics is not designed to represent the outline of a course that might lead to the mathematics SL or mathematics HL course. Instead, it lists the knowledge, together with the syllabus content, that is essential to successful completion of the mathematics SL or mathematics HL course.

Students must be familiar with SI (*Système International*) units of length, mass and time, and their derived units.

Topics

Number and algebra

Routine use of addition, subtraction, multiplication and division using integers, decimals and fractions, including order of operations.

Example: $2(3+4\times 7)=62$.

Simple positive exponents.

Examples: $2^3=8$; $(-3)^3=-27$; $(-2)^4=16$.

Simplification of expressions involving roots (surds or radicals).

Examples: $\sqrt{27}+\sqrt{75}=8\sqrt{3}$; $\sqrt{3}\times\sqrt{5}=\sqrt{15}$.

Prime numbers and factors, including greatest common factors and least common multiples.

Simple applications of ratio, percentage and proportion, linked to similarity.

Definition and elementary treatment of absolute value (modulus), $|a|$.

Rounding, decimal approximations and significant figures, including appreciation of errors.

Expression of numbers in standard form (scientific notation), that is, $a\times 10^k$, $1\le a<10$, $k\in\mathbb{Z}$.

Concept and notation of sets, elements, universal (reference) set, empty (null) set, complement, subset, equality of sets, disjoint sets. Operations on sets: union and intersection. Commutative, associative and distributive properties. Venn diagrams.

Number systems: natural numbers; integers, $\mathbb{Z}$; rationals, $\mathbb{Q}$, and irrationals; real numbers, $\mathbb{R}$.

Intervals on the real number line using set notation and using inequalities. Expressing the solution set of a linear inequality on the number line and in set notation.

The concept of a relation between the elements of one set and between the elements of one set and those of another set. Mappings of the elements of one set onto or into another, or the same, set. Illustration by means of tables, diagrams and graphs.

Basic manipulation of simple algebraic expressions involving factorization and expansion.

Examples: $ab+ac=a(b+c)$; $(a\pm b)^2=a^2+b^2\pm 2ab$; $a^2-b^2=(a-b)(a+b)$;
$3x^2+5x+2=(3x+2)(x+1)$; $xa-2a+xb-2b=(x-2)(a+b)$.

Rearrangement, evaluation and combination of simple formulae. Examples from other subject areas, particularly the sciences, should be included.

Presumed knowledge for mathematics SL and HL

The linear function $x \mapsto ax + b$ and its graph, gradient and *y*-intercept.

Addition and subtraction of algebraic fractions with denominators of the form $ax + b$.

Example: $\frac{2x}{3x-1} + \frac{3x+1}{2x+4}$.

The properties of order relations: <, =, >, =.

Examples: $a > b, c > 0 \Rightarrow ac > bc$; $a > b, c < 0 \Rightarrow ac < bc$.

Solution of equations and inequalities in one variable, including cases with rational coefficients.

Example: $\frac{3}{7} - \frac{2x}{5} = \frac{1}{2}(1-x) \Rightarrow x = \frac{5}{7}$.

Solution of simultaneous equations in two variables.

Geometry

Elementary geometry of the plane including the concepts of dimension for point, line, plane and space. Parallel and perpendicular lines, including $m_1 = m_2$, and $m_1 m_2 = -1$. Geometry of simple plane figures. The function $x \mapsto ax + b$: its graph, gradient and *y*-intercept.

Angle measurement in degrees. Compass directions and bearings. Right-angle trigonometry. Simple applications for solving triangles.

Pythagoras' theorem and its converse.

The Cartesian plane: ordered pairs (x, y), origin, axes. Mid-point of a line segment and distance between two points in the Cartesian plane.

Simple geometric transformations: translation, reflection, rotation, enlargement. Congruence and similarity, including the concept of scale factor of an enlargement.

The circle, its centre and radius, area and circumference. The terms "arc", "sector", "chord", "tangent" and "segment".

Perimeter and area of plane figures. Triangles and quadrilaterals, including parallelograms, rhombuses, rectangles, squares, kites and trapeziums (trapezoids); compound shapes.

Statistics

Descriptive statistics: collection of raw data, display of data in pictorial and diagrammatic forms (for example, pie charts, pictograms, stem and leaf diagrams, bar graphs and line graphs).

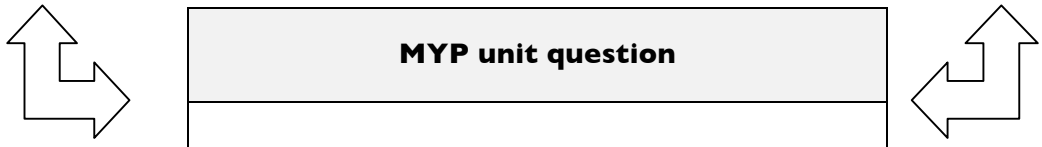
Calculation of simple statistics from discrete data, including mean, median and mode.

MYP unit planner

Unit title	
Teacher(s)	
Subject and grade level	
Time frame and duration	

Stage 1: Integrate significant concept, area of interaction and unit question

Area of interaction focus Which area of interaction will be our focus? Why have we chosen this?		**Significant concept(s)** What are the big ideas? What do we want our students to retain for years into the future?



MYP unit question

Assessment What task(s) will allow students the opportunity to respond to the unit question? What will constitute acceptable evidence of understanding? How will students show what they have understood?
Which specific MYP objectives will be addressed during this unit?
Which MYP assessment criteria will be used?

Stage 2: Backward planning: from the assessment to the learning activities through inquiry

Content

What knowledge and/or skills (from the course overview) are going to be used to enable the student to respond to the unit question?

What (if any) state, provincial, district, or local standards/skills are to be addressed? How can they be unpacked to develop the significant concept(s) for stage 1?

Approaches to learning

How will this unit contribute to the overall development of subject-specific and general approaches to learning skills?

Learning experiences

How will students know what is expected of them? Will they see examples, rubrics, templates?

How will students acquire the knowledge and practise the skills required? How will they practise applying these?

Do the students have enough prior knowledge? How will we know?

Teaching strategies

How will we use formative assessment to give students feedback during the unit?

What different teaching methodologies will we employ?

How are we differentiating teaching and learning for all? How have we made provision for those learning in a language other than their mother tongue? How have we considered those with special educational needs?

Resources

What resources are available to us?

How will our classroom environment, local environment and/or the community be used to facilitate students' experiences during the unit?

Ongoing reflections and evaluation

In keeping an ongoing record, consider the following questions. There are further stimulus questions at the end of the "Planning for teaching and learning" section of *MYP: From principles into practice*.

Students and teachers

What did we find compelling? Were our disciplinary knowledge/skills challenged in any way?

What inquiries arose during the learning? What, if any, extension activities arose?

How did we reflect—both on the unit and on our own learning?

Which attributes of the learner profile were encouraged through this unit? What opportunities were there for student-initiated action?

Possible connections

How successful was the collaboration with other teachers within my subject group and from other subject groups?

What interdisciplinary understandings were or could be forged through collaboration with other subjects?

Assessment

Were students able to demonstrate their learning?

How did the assessment tasks allow students to demonstrate the learning objectives identified for this unit? How did I make sure students were invited to achieve at all levels of the criteria descriptors?

Are we prepared for the next stage?

Data collection

How did we decide on the data to collect? Was it useful?

Figure 12

MYP unit planner

Mathematics assessment criteria

Please note that the assessment criteria in this guide are for first use in **final assessment** in 2008 for southern hemisphere schools and 2009 for northern hemisphere schools.

The following assessment criteria have been established by the IB for mathematics in the MYP. All final assessment in the final year of the MYP must be based on these assessment criteria even if schools are not registering students for IB-validated grades and certification.

Criterion A	Knowledge and understanding	Maximum 8
Criterion B	Investigating patterns	Maximum 8
Criterion C	Communication in mathematics	Maximum 6
Criterion D	Reflection in mathematics	Maximum 6

- For each assessment criterion, a number of band descriptors are defined. These describe a range of achievement levels, with the lowest represented as 0.
- The criteria are not equally weighted.
- The descriptors concentrate on positive achievement, although failure to achieve may be included in the description for the lower levels.

Detailed descriptions of the assessment criteria and band descriptors follow.

Criterion A: Knowledge and understanding

Maximum: 8

Knowledge and understanding are fundamental to studying mathematics and form the base from which to explore concepts and develop skills. This criterion expects students to use their knowledge and to demonstrate their understanding of the concepts and skills of the prescribed framework in order to make deductions and solve problems in different situations, including those in real-life contexts.

This criterion examines to what extent the student is able to:

- know and demonstrate understanding of the concepts from the five branches of mathematics (number, algebra, geometry and trigonometry, statistics and probability, and discrete mathematics)
- use appropriate mathematical concepts and skills to solve problems in both familiar and unfamiliar situations, including those in real-life contexts
- select and apply general rules correctly to solve problems, including those in real-life contexts.

Assessment tasks for this criterion are likely to be class tests, examinations, real-life problems and investigations that may have a variety of solutions.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors given below.
1–2	The student **attempts** to make deductions when solving **simple** problems in **familiar** contexts.
3–4	The student **sometimes** makes **appropriate** deductions when solving **simple and more-complex** problems in **familiar** contexts.
5–6	The student **generally** makes **appropriate** deductions when solving **challenging** problems in a **variety** of **familiar** contexts.
7–8	The student **consistently** makes **appropriate** deductions when solving **challenging** problems in a **variety** of contexts including **unfamiliar** situations.

Notes

1. Context: the situation and the parameters given to a problem.
2. Unfamiliar situation: challenging questions or instructions set in a new context in which students are required to apply knowledge and/or skills they have been taught.
3. Deduction: reasoning from the general to the particular/specific.

Criterion B: Investigating patterns

Maximum: 8

Students are expected to investigate a problem by applying mathematical problem-solving techniques, to find patterns, and to describe these mathematically as relationships or general rules and justify or prove them.

This criterion examines to what extent the student is able to:

- select and apply appropriate inquiry and mathematical problem-solving techniques
- recognize patterns
- describe patterns as relationships or general rules
- draw conclusions consistent with findings
- justify or prove mathematical relationships and general rules.

Assessment tasks for this criterion should be mathematical investigations of some complexity, as appropriate to the level of MYP mathematics. Tasks should allow students to choose their own mathematical techniques to investigate problems, and to reason from the specific to the general. Assessment tasks could have a variety of solutions and may be set in real-life contexts. Teachers should clearly state whether the student has to provide a justification or proof.

Teachers should include a good balance between tasks done under test conditions and tasks done at home in order to ensure the development of independent mathematical thinking.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors given below.
1–2	The student **applies, with some guidance**, mathematical problem-solving techniques to recognize **simple** patterns.
3–4	The student **selects and applies** mathematical problem-solving techniques to recognize patterns, and **suggests** relationships or general rules.
5–6	The student **selects and applies** mathematical problem-solving techniques to recognize patterns, **describes** them as relationships or general rules, and **draws conclusions** consistent with findings.
7–8	The student **selects and applies** mathematical problem-solving techniques to recognize patterns, **describes** them as relationships or general rules, **draws conclusions** consistent with findings, and **provides justifications or proofs**.

Notes

1. Pattern: the underlining order, regularity or predictability between the elements of a mathematical system. To identify pattern is to begin to understand how mathematics applies to the world in which we live. The repetitive features of patterns can be identified and described as relationships or generalized rules.
2. Justification: a clear and logical mathematical explanation of **why** the rule works.
3. Proof: a mathematical demonstration of the truth of a given proposition.

Criterion C: Communication in mathematics

Maximum: 6

Students are expected to use mathematical language when communicating mathematical ideas, reasoning and findings—both orally and in writing.

This criterion examines to what extent the student is able to:

- use appropriate mathematical language (notation, symbols, terminology) in both oral and written explanations
- use different forms of mathematical representation (formulae, diagrams, tables, charts, graphs and models)
- communicate a complete and coherent mathematical line of reasoning using different forms of representation when investigating complex problems.

Students are encouraged to choose and use appropriate ICT tools such as graphic display calculators, screenshots, graphing, spreadsheets, databases, drawing and word-processing software, as appropriate, to enhance communication.

Assessment tasks for this criterion are likely to be real-life problems, tests, examinations and investigations. Tests and examinations that are to be assessed against criterion C must be designed to allow students to show complete lines of reasoning using mathematical language.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors given below.
1–2	The student shows **basic** use of mathematical language **and/or** forms of mathematical representation. The lines of reasoning are **difficult to follow**.
3–4	The student shows **sufficient** use of mathematical language **and** forms of mathematical representation. The lines of reasoning are **clear** though not always **logical** or **complete**. The student moves between different forms of representation **with some success**.
5–6	The student shows **good** use of mathematical language **and** forms of mathematical representation. The lines of reasoning are **concise**, **logical** and **complete**. The student moves **effectively** between different forms of representation.

Notes

1. Mathematical language: the use of notation, symbols, terminology and verbal explanations.
2. Forms of mathematical representation: refers to formulae, diagrams, tables, charts, graphs and models, used to represent mathematical information.

Criterion D: Reflection in mathematics

Maximum: 6

Reflection allows students to reflect upon their methods and findings.

This criterion examines to what extent the student is able to:

- explain whether his or her results make sense in the context of the problem
- explain the importance of his or her findings in connection to real life
- justify the degree of accuracy of his or her results where appropriate
- suggest improvements to the method when necessary.

Assessment tasks are most likely to be investigations and real-life problems. Generally these types of tasks will provide students with opportunities to use mathematical concepts and skills to solve problems in real-life contexts.

Achievement level	Level descriptor
0	The student does not reach a standard described by any of the descriptors given below.
1–2	The student **attempts** to explain whether his or her results make sense in the context of the problem. The student **attempts to describe** the importance of his or her findings in connection to real life.
3–4	The student **correctly but briefly explains** whether his or her results make sense in the context of the problem and **describes** the importance of his or her findings in connection to real life. The student **attempts** to justify the degree of accuracy of his or her results where appropriate.
5–6	The student **critically explains** whether his or her results make sense in the context of the problem and provides a **detailed explanation** of the importance of his or her findings in connection to real life. The student **justifies** the degree of accuracy of his or her results where appropriate. The student **suggests improvements** to the method when necessary.

Notes

1. Describe: present an account without providing reasons or explanations.
2. Explain: give a detailed account including reasons, causes or justifications. Explanations should answer the questions “why” and “how”.

Determining the final grade

This section explains the process by which a student's overall achievement level (in terms of the assessment criteria) is converted to a single grade.

1. Collecting the information

Teachers will use assessment tasks to make judgments of their students' performance against the assessment criteria at intervals during the final year in the subject. Many of the assessment tasks will allow judgments of levels to be made with regard to more than one criterion.

For the purposes of final assessment, teachers **must** ensure that, for each student, they make **several judgments against each criterion**. This can be achieved by using some kinds of assessment task more than once, or by incorporating other types of assessment activity. MYP mathematics has **four** criteria and so **at least eight** judgments (two per criterion) must be made for each student in the final year for the purposes of final assessment. However, as more-complex tasks will allow students to be assessed against several criteria, final assessment may rest on a limited number of tasks.

Important: If more than one teacher is involved in one subject for a single year group, the school must ensure **internal standardization** is used to provide a common system for the application of the assessment criteria to each student. In joint assessment, internal standardization is best achieved by:

- the use of common assessment tasks
- shared assessment between the teachers
- regular contact between the teachers.

In certain schools, students may be grouped according to ability within the same subject (for example, standard and extended levels in mathematics). In such cases, the teachers' final assessment of student performance across all groups or levels must be based on a **consistent application of the assessment criteria to all students**. A different standard should not be applied to different groups or levels.

2. Making a final judgment for each criterion

When the judgments on the various tasks have been made, teachers will be in a position to establish a final profile of achievement for each student by determining the **single most appropriate level for each criterion**. Where the judgments for a criterion differ for specific assessment tasks, the teacher must decide which level best represents the student's final standard of achievement.

Important: Teachers should not average the levels gained in year 5 for any given criterion. Students can develop academically right up to the end of the programme, and teachers must make a professional judgment (that is also supported by work completed) as to which level best corresponds to a student's general level of performance for each of the criteria towards the end of the programme.

3. Determining the final criterion levels total

The final levels for each criterion must then be added together to give a **final criterion levels total** for mathematics for each student. In mathematics, students have the opportunity to gain a maximum level of 8 for criteria A and B, and a maximum level of 6 for criteria C and D. Therefore the maximum final criterion levels total for mathematics will be 28.

The final criterion levels total is the total that will be submitted to the IB via IBIS (IB information system) for those schools that have registered students to receive IB-validated grades.

4. Determining the final grade for mathematics

Grade boundaries must be applied to the criterion levels totals to decide the final grade for each student.

Please see the *MYP coordinator's handbook* for the table of grade boundaries for mathematics.

All MYP subjects receive final grades in the range from 1 (lowest) to 7 (highest) on the IB record of achievement, where students have been registered for IB-validated grades. The general MYP grade descriptors describe the achievement required for the award of the subject grade. After using the conversion table to determine a student's final mathematics grade, teachers should check the general grade descriptor table to ensure that the description equally reflects the student's achievement.

Schools requiring **IB-validated grades** are required to use **only** the published MYP subject-specific criteria and grade boundaries as a basis for the final results that they submit to the IB (both for moderation and as final assessment for certification).

Other schools (those not requiring IB-validated grades) will use the published criteria together with any additional criteria that they have developed independently, and report internally to students and parents. These schools may decide on their own grade boundaries (if using published and additional criteria), or use the boundaries published by the IB.

Assessment rubric template

Criterion A: Knowledge and understanding

Criterion A	Making deductions	Type of problem	Context
1-2	Attempts	Simple	Familiar
3-4	Sometimes appropriate	More-complex	Familiar
5-6	Generally appropriate	Challenging	Familiar
7-8	Consistently appropriate	Challenging	Unfamiliar

Criterion B: Investigating patterns

Criterion B	Patterns and relationships	Problem-solving techniques
1-2	Recognizes simple patterns	With guidance
3-4	Suggests relationships or general rules	Selects and applies
5-6	Draws conclusions consistent with findings	Selects and applies
7-8	Provides justifications or proofs	Selects and applies

Criterion C: Communication in mathematics

Criterion C	Mathematical language and forms of mathematical representation	Lines of reasoning
1-2	Basic use of one and/or the other	Difficult to follow
3-4	Sufficient use of both, moves between them with some success	Clear but not always logical or complete
5-6	Good use of both, moves between them effectively	Concise, logical and complete

D: Reflection in mathematics

Criterion D	Results in the context of the problem	Importance of findings in connection with real-life	Degree of accuracy of results	Method
1-2	Attempts to explain	Attempts to describe	----------	----------
3-4	Correctly but briefly explains	Describes	Attempts to justify	----------
5-6	Critically explains	Detailed explanation	Justifies	Suggests improvements

Teacher's task – Should we start melting our coins?

In this investigation you are going to investigate if we should start melting our coins?

1. Read the Financial Times article, highlighting the most important facts.

2. When was the article written?

3. Use the information in the article to find out:
   a) the mass of one coin?
   b) the mass of zinc in one coin?
   c) the mass of copper in one coin?

4. Use the metal prices in the article to confirm Ed Yardeni's words.

5. Check the current metal prices and find out if it is now worth melting the 1cent US coins.

6. What are the metals used to make the Euro coins? Why?

7. Investigate if it is worth melting them? Collect the relevant information.

8. What degree(s) of accuracy did you use? Why?

9. What is your prediction for the near future; will the value of the metal overcome the money value? Justify your answer.

10. a) How do you know your methods are correct?
    b) How do you know your findings are reliable?
    c) Was the ratio metal value/face value of the coin the same for the euro coins tested? Why?
    d) Can you suggest improvements to the method?

12. Write a newspaper article (title, introduction, body of the investigation and conclusion) to report your findings, it should be clear to someone who did not read the Financial Times article or saw the questions. Cite all sources to give credit to your article.

High metal prices mean melting coins could start making cents

Financial Times
By Kevin Morrison in London
Published: April 11 2006 03:00

It could soon be worth Americans melting down their pennies for scrap, if zinc and copper prices continue their current rate of increase.

Copper prices have risen 30 per cent so far this year, and zinc is up 55 per cent - a rise of about $550 a tonne in a little more than three weeks.

A rise by the same magnitude would make the metal content in the US one cent coin worth more than its face value.

There are 160 pennies - also known as a one cent coin – in a pound (lb), worth a face value of $1.60. But – with each penny made of 97.50 per cent zinc and 2.5 per cent copper – based on current prices, the metal value is worth about $1.36. Therefore another 25 cents-a-pound rise in zinc, or about $551 a tonne, would see the metal value of the US penny worth more than the monetary value.

Market analysts do not rule out such a rise in the zinc price. “Zinc prices have already risen further than what most people had ever thought but that is not to say there cannot be further gains,” said Ingrid Sternby, analyst at Barclays Capital. “Could we see another $500 on zinc? It’s possible she said.”

However, Ed Yardeni, chief investment strategist at US fund manager Oak Associates, who drew attention to the penny’s rising scrap value said: “Don’t bother melting the pennies just yet. However at the rate that metal prices have soared since last year, there might be soon an arbitrage play between pennies and zinc.”

Copper and zinc yesterday extended their record-breaking run to reach new highs of $5,960 and $2,930 a tonne respectively. But Simon Hayley, senior international economist at Capital Economics, said it would start to fade by the end of the year.

Morrison, K. 11 April 2006. “High metal prices mean melting coins could start making cents”. *Financial Times*

Student work – Should we start melting our coins?

What is the mass of one penny, if their are 160 pennies in 1lb (pound)

160 pennies = 1 pound (453.59 g)
1 penny = x

$$\frac{160}{453.59} = \frac{1}{x}$$

$$x = \frac{453.59}{160}$$

x = 2.835 g the mass of one penny

What is the mass of zinc in one coin?

97.5% of zinc in one coin
97.5% of 2.835g

$$\frac{97.5}{100} = \frac{x}{2.835}$$

$$x = \frac{97.5 \times 2.835}{100}$$

x= 2.764125 g of zinc in one penny

What is the mass of copper in one coin?

2.5% of copper in one coin
2.5% of 2.835g

$$\frac{2.5}{100} = \frac{x}{2.835}$$

$$x = \frac{2.5 \times 2.835}{100}$$

x= 0.070875 g of copper in one penny

Use the metal prices in the article to confirm Ed Yardeni's words.

The value of a tonne of zinc is $2930, and the value of a tonne of copper is $ 5960. To find the value of one gram of each you divide by 1,000,000.

Zinc: $2930/ 1,000,000 = $ 0.00293/g

Copper: $5960/ 1,000, 000 = $ 0.00596/g

You now times the mass of the metal in a coin by the cost of the metal per gram to find it's metal value.

2.76412 5$\times$ 0.00293= $ 0.0080988863

0.070875 $\times$ 0.00596 = 4.22415$\times 10^{-4}$ = $ 0.000422415

0.0080988863+ 0.000422415 = $ 0.0085213013 the cost of the metal per penny

$ 0.0085213013 $\times$ 160 = $ 1.3634 in 160 coins

In the article Ed Yaderni states that the metal value is worth about $1.36, which is correct, however a penny is worth $0.01 so it is not worth yet melting our coins because the outcome is not superior to the value of the coin.

If the metal value keeps rising, and comes to be greater than the face value then it will be worth melting our coins.

Checking the current metal prices and find out if it is now worth melting the 1cent US coins.

To work out if it's worth melting I cent in us coins you have to do the following:

Metal price of coin = price of zinc per gram $\times$ mass of zinc in coin +
price of copper per gram $\times$ mass of copper in coin

a) Zinc: current price of zinc is $3375/tonne

$0.003375 \times 2.76 = \$0.009315$ price of zinc in a penny

b) Copper: current price is $7600/tonne

$0.0076 \times 0.07 = \$0.000532$ price of copper in a penny

$0.009315 + 0.000532 = \$0.009847$ metal price in one penny

It is not worth melting the 1cent Us coins yet.

What degree (s) of accuracy did I use? Why?

In question four I choose to use the degree of accuracy to 3 decimal places, to get and impartial and more accurate answer. Too much rounding up will increase my results.

What are the metals used to make the Euro coins? Why?

The metals used to make a €0.50 or a €1.00 coin, are:

50 cents Euro (€0,50)
Diameter: 24,25mm
weight: 7,80g
Composition: northic gold (Cu 89%; Al 5%; Zn 5%; Sn 1%).

1 Euro (€ 1)
Diameter: 23,25mm
Weight: 7,50g
Composition: crown (3,79g): "níquelado" brass (Cu 75%; Zn 20%; Ni 5%); nucleus (3,71g): nickel coated with "cuproníquel" (Cu 75%; Ni 25%).

As you can notice both coins are mostly made of copper, this is because copper is very resistant. It is said that copper can last up to 30 years, we also know that copper is very flexible so it is easier to carve images, and make smooth edges.

Is it worth melting the Euro coins? Collect relevant information.

50 cents coins:

mass : 7.80g

Copper: 89% of 7.80g

$$\frac{7.80}{100} = \frac{x}{89}$$

$$x = \frac{7{,}80 \times 89}{100}$$

X= 6.942g of copper in a 50 cent coin

Zinc and Aluminium

5% of 7.80g

$$\frac{7.80}{100} = \frac{x}{5}$$

$$x = \frac{7.80 \times 5}{100}$$

x= 0.39g of zinc and aluminium

Tin:

1% of 7,80g

$$\frac{7.80}{100} = \frac{x}{1}$$

$$x = \frac{7.80 \times 1}{100}$$

x= 0.078g of tin in a 50 cent euro coin

These are the prices I found on metalprices.com:

Aluminium = 2481 US dollar / tonne

Zinc = 3375 US dollar / tonne

Copper = 7600 US dollar / tonne

Tin = 9115 US dollar / tonne

1 US dollar = € 0.7877

Now I need to convert the metal prices that are in US dollar/tonne into Euro/gram

Aluminium:

$2481 × 0.7877= € 1954.28 / tonne

1954.28/1,000,000= € 0.00195428

= € 0.0020/ grams

Zinc:

3375 * 0.7877g = € 2658.49 / tonne

2658.49/ 1,000,000= € 0.00265849

= € 0.0027/gram

Copper:

7600* 0.7877 = € 5986.52

5986.52/1,000,000= € 0.00598652

= €0.0060/gram

Tin:

9115 × 0.7877 = € 7179.89 / tonne

7179.89/ 1,000,000 = € 0.00717989

= € 0.0072/grams

Finally to find the metal price of the coin:

Metal price of coin = price of metal per gram × mass of metal in coin +

price of metal per gram × gram of metal in coin

copper = 0.0060 × 6.942 = € 0.041652

aluminium = 0.0020 × 0.39 = € 0.00078

zinc = 0.0027 × 0.39 = € 0.001053

tin = 0.0072 × 0.07 = € 0.0005616

Metal price of the coin:

0.041652 +0.00078+0.001053+0.0005616= € 0.0440466

If the metal price of a 50 cent coin doesn't reach it's value then the metal price of a one euro coin certainly doesn't reach it, being 0.50 a lower value.

What degree (s) of accuracy did you use? Why?

In question 7, I started by leaving my answers to 2 decimal places but after when using them I left the results as they came. That is because in this case 2 decimal places seemed the best way to present my answer without excluding any important numbers. Later a I didn't round my answers because I was not dealing with "big" numbers and to have a better idea of how much the metal price of the coin really is.

What is your prediction for the near future? Will the value of the metal overcome the money value? Justify your answer.

Metal prices rise fast, but they are not constant so they have their ups and downs. As they can increase a lot in the future days they can also decrease rapidly.

When coins are created they are thought of in everyway and the composition of the coin was probably studied in order of its metals not reaching greater value than it self.

How do you know your methods are correct?

My methods are correct because to calculate the metal price of a coin you need to multiply the metal price by the metal in the coin and then add.

How do you know your results are reliable?

My results aren't 100% reliable because they were obtained with minor calculations, and the metal prices keep increasing and decreasing everyday and hour. However my conclusions should be reliable, because the metal prices changes wouldn't affect the results marjory. The website I used to collect my information was used by most of my colleagues so I believe it must be a secure source.

Newspaper article (title introduction, body of the investigation and conclusion) to report my findings, it should be clear to someone who did not read the Financial Tines article or saw the questions. Cite all sources to give credit to your article.

A group of students from the international school of , have lately been studying for a school assessment if it is worth Europeans with the Euro as their current money melting it.

This is based on an article by Kevin Morrison which was published on the 11th of April 2006 in the Financial Times, where he states that the metal price for some metals used in the American penny coins could come to rise so much that it will overcome the face value of the coin. And it will then be worth Americans melting down their pennies for scrap.

However, the metal prices are not higher than the coin value yet.

Based on that the year 11 mathematics students were asked to investigate if the same happens for euro coins.

From their results they conclude that it is certainly not worth Europeans melting their coins, at least now and in a near future.

An example of that could be the fact that if they melted a €0.50 coin, they would only sell it by € 0.04 cents, like that losing money.

The metal prices keep rising and will keep rising due to "Robust western world growth outlook, Chinese demand, continuing supply constraints, further fall in inventories from already low levels, investment appetite among funds and poor support from currencies are combining to keep the base metals market hot." *(http://www.thehindubusinessline.com/bline/2005/03/27/stories/2005032700110400.htm)

This means that if this continues in future the metal prices may rise so much it will be possible to profit from melting euro coins and selling them as metals. Though the prices would have to go up a lot.

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Moderator comments – Should we start melting our coins?

Example 6: Real-life problem Melting coins

Standard mathematics

Rea-life problem

Branch: Number

MYP year: 5

Criterion	A	B	C	D
Level achieved	6	-	3	2

Background

For this real-life problem students needed to have covered number concepts such as approximations, scientific notation, ratios, proportion, and decimal system they also needed to have a general awareness of system of the major imperial units.

Students were given an instruction sheet and an article from a financial newspaper to carry out this real-life problem. Students had to read and analyse the article which dealt with the rising price of metals and how, in a near future, the metal value of the US dollar one cent coin would overcome its face value. Students were expected to use a variety of number concepts to find out if the claim of the article was also true for any Euro coins. As part of the investigation, students also had to make future predictions. Finally students had to communicate their findings in the form of a newspaper article. The best articles of the class were to be published in the school newspaper.

Students had one double lesson of 90 minutes to work on the task and one week of homework. Students had access to the Internet and other resources such as newspapers, magazines, books. Students had to find information on the metal composition of the Euro coins, the price of the metals, the Euro–US Dollar exchange rate and the conversion rates between the imperial and decimal systems.

This investigation contributed to raising students' awareness of the possible application of mathematics to real-life situations. It also gave students an idea of the fluctuations of the stock market, how and why the price of the metals and exchange rates change as part of the economy and financial markets.

This investigation was open-ended as different students found different data so each project was quite original. It was an appropriate task for year 5 students because even though the concepts were not too difficult, the task required students to understand the stock market and to find, select and analyse information from a range of sources, make reasoned deductions and critically make a prediction for the future.

Assessment

Criterion A: knowledge and understanding

Maximum 8

Achievement level	Descriptor
5–6	The student **generally** makes **appropriate** deductions when solving **challenging** problems in a **variety** of **familiar** contexts.

This work achieved level 6 because the student:

- makes appropriate deductions when solving problems even when the situation was unfamiliar
- shows a good understanding of number in its different representations, selecting and using appropriate concepts to solve problems.

The student would have achieved a higher level if he had:

- tested for other coins, such as coins with a lower face value instead of choosing the Euro 50 cent coin and making unsupported assumptions about the Euro 1 coin
- represented the metal value as a percentage of the face value.

Criterion C: communication in mathematics

Maximum 6

Achievement level	Descriptor
3–4	The student shows **sufficient** use of mathematical language **and** forms of mathematical representation. The lines of reasoning are **clear** though not always **logical** or **complete**. The student moves between different forms of representation **with some success**.

This work achieved level 3 because the student:

- shows good use of mathematical language and units throughout the problem
- communicates his findings using a clear and logical lines of reasoning

The student would have achieved a higher level if he had also used other forms of mathematical representation such as a table or graph to communicate his findings

Criterion D: reflection in mathematics

Maximum 6

Achievement level	Descriptor
1–2	The student **attempts** to explain whether his or her results make sense in the context of the problem. The student **attempts to describe** the importance of his or her findings in connection to real life.

This work achieved level 2 because the student:

- makes an attempt to justify the degree of accuracy chosen for the answers but it is not supported by the results
- briefly explains the fact that it is not yet worth melting the coins

The student would have achieved a higher level if he had:

- calculated the ratio face value/metal value for the coins tested and commented on the difference
- justified the degree of accuracy used correctly

EXPLANATION OF MYP TASK TWO

WRITTEN REPORT OF AN INVESTIGATION

Students wrote this in class under test conditions. They had to email their work at the end of class each day. They had a total of 2.5 hours to complete the tasks and were given the reflection to take home and hand in the next day.

Marking Scheme

- All MYP criterion based assessment, focusing on:
    - Criterion B: Investigating Patterns
    - Criterion C: Communication in Mathematics
    - Criterion D: Reflection in Mathematics.

Criterion B: Investigating Patterns

- Students are to determine the general rules associated with transformations of quadratic functions
- Open-ended to allow all levels of ability – students who are more capable can combine all transformations and look at more advanced functions, such as cubics, to see if the same general rules they found to exist for parabolas, are true for higher order functions
- They are asked to validate their generalizations and apply them to a particular question to ensure they are correct in all cases
- Students are asked to justify/validate their general rule that they have determined.

Criterion C: Communication in Mathematics

- Students are asked to summarize their findings and general rules in a one page report, illustrating all of the key information, using correct symbols and terminology
- Students must show evidence of moving between the algebraic and graphic forms of quadratic functions.

Criterion D: Reflection in Mathematics

- Students are asked to critically explain the results obtained in the context of the problem.
- Students are asked when to apply their general rule, why it is presented in that particular form and how they can connect their findings to real life.
- Students were asked to suggest improvements to their problem-solving methodology.
- They are also asked to reflect on the use of technology, which was instrumental in allowing them to focus on determining the rule.

Part Two: Summary of generalizations

Using all of the information you discovered in Part One, create a one to two page summary of all of the different types of transformations, detailing your generalizations about each type and a diagram to illustrate your general rules.

Since you are not handing in the answers to part one separately, you must make sure that you include all of the necessary points that you determined in your answers to the questions in part one.

This is part of your communication mark, so make sure you clearly explain your reasoning, using appropriate terminology and symbols.

All of your diagrams are to be created using graphing software

Question five: Translations

$y = x^2$

$y = (x-1)^2$

$y = (x+3)^2$

(a) Graph the above functions on the same set of axes

(b) Copy and paste the above functions in your Word document.

(c) What is the axis of symmetry and vertex of each function?

(d) What do the graphs have in common?

(e) What effect does h have on the graph $y = (x - h)^2$ if $h > 0$?

(f) What effect does h have on the graph $y = (x - h)^2$ if $h < 0$?

(g) Write a general statement about the effect of changes in h on the graph $y = x^2$

Question six: Combinations

$y = x^2$

$y = -2(x-4)^2 + 3$

(a) Graph the above functions on the same set of axes

(b) Copy and paste the above functions in your Word document.

(c) What is the axis of symmetry and vertex of each function?

(d) How do the constraints -2, 4 and 3 affect the graph $y = x^2$ (do a step by step transformation from $y = x^2$ to $y = -2(x - 4)^2 + 3$)

(e) Describe, in general, what effect each constraint (a, h and k) has on the graph $y = a(x - h)^2 + k$ compared to $y = x^2$?

Investigate how a, h and k affect the graph of $y = x^2$ in the equation of $y = a(x - h)^2 + k$.

Part One: Transformations of Quadratic Functions

Question one: Dilations

$y = x^2$

$y = 2x^2$

$y = 4x^2$

(a) Graph the above functions on the same set of axes.

(b) Copy and paste the above functions in your Word document.

(c) What is the axis of symmetry and vertex of each function?

(d) What do the graphs have in common?

(e) What happens to the graph of $y = x^2$ when x is multiplied by a positive number greater than 1?

(f) Write a general statement about the effect of changes in *a* on the graph $y = ax^2$ when $a > 1$?

Question two: Dilations

$y = x^2$

$y = \frac{1}{2}x^2$

$y = \frac{1}{4}x^2$

(a) Graph the above functions on the same set of axes:

(b) Copy and paste the above functions in your Word document.

(c) What is the axis of symmetry and vertex of each function?

(d) What do the graphs have in common?

(e) What happens to the graph of $y = x^2$ is multiplied by a positive number less than 1 but larger than zero?

(f) Write a general statement about the effect of changes in a on the graph $y = ax^2$ when $1>a>0$?

Question three: Reflections

$y = x^2$

$y = -2x^2$

$y = -\frac{1}{3}x^2$

(a) Graph the above functions on the same set of axes:

(b) Copy and paste the above functions in your Word document.

(c) What is the axis of symmetry and vertex of each function?

(d) What do the graphs have in common?

(e) What happens to the graph of $y = x^2$ is multiplied by a negative number?

(f) Write a general statement about the effect of changes in a on the graph $y = ax^2$ when $a<0$?

Question four: Translations

$y = x^2$

$y = x^2 + 2$

$y = x^2 - 4$

(a) Graph the above functions on the same set of axes

(b) Copy and paste the above functions in your Word document.

(c) What is the axis of symmetry and vertex of each function?

(d) What do the graphs have in common?

(e) What effect does k have on the graph $y = x^2 + k$ if $k > 0$?

(f) What effect does k have on the graph $y = x^2 + k$ if $k < 0$?

(g) Write a general statement about the effect of changes in k on the graph $y = x^2$

Investigate how a, h and k affect the graph of $y = x^2$ in the equation of $y = a(x - h)^2 + k$.

Part Three: Examples to validate your general rules

Create an illustration of each of the following parabolas, using $y = x^2$ as a reference on each illustration.

Clearly explain **each step** that is required to transform the parabola $y = x^2$ into each of the following 4 separate parabolas.

For each, clearly label:

a) the axis of symmetry (equation of the line)

b) the vertex

c) the domain and range

$$y = -\frac{1}{4}x^2$$

$$y = 2x^2 - 10$$

$$y = -4(x+3)^2$$

$$y = \tfrac{1}{2}(x-2)^2 + 8$$

Part Four: Reasoning and Justification

Answer the following questions in sentence form, using examples where suitable to illustrate your point:

How did the use of technology help you during the course of this investigation?

Can you think of any improvements that you could have made to your problem solving methodology during this assignment?

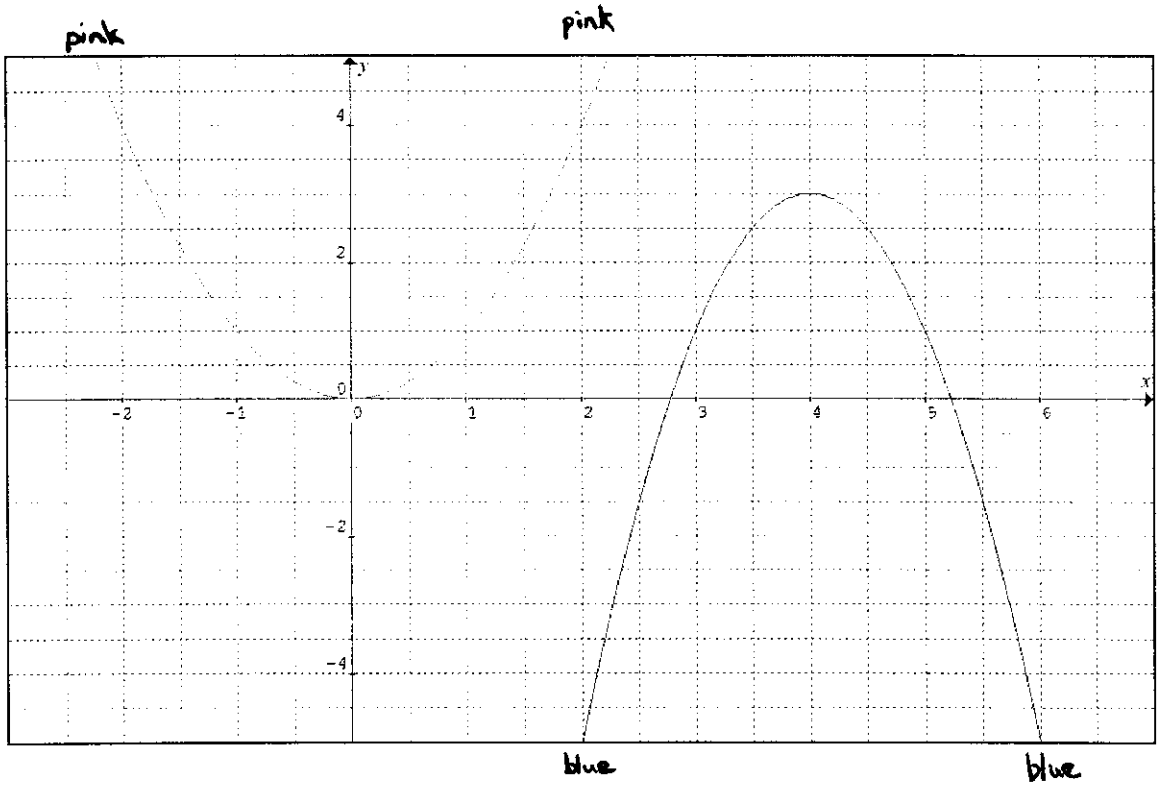
Why is the generalized form of the equation expressed as $y = a(x-h)^2 + k$?

How do you think this formula could be used in real life applications?

How do you know your generalizations about transformations are correct? Justify/validate your general rules.

Can these generalizations be applied to other functions? Explain your reasoning.

ANSWER KEY TO TRANSFORMATIONS ASSIGNMENT



The pink function is $y = x^2$ and the blue function is $y = -2(x-4)^2 + 3$

Transformations that have taken place to the basic $y = x^2$ to get $y = -2(x-4)^2 + 3$

transformation	number	explanation
dilation	2	▪ proper mathematical terminology is a vertical stretch ▪ would accept *y* value increases at a faster rate than it did in the original function ▪ in this case, the function has stretched vertically by a factor of 2
reflection	Negative in front of the 2	▪ reflection in the x axis ▪ causes the function to open downwards instead of upwards ▪ function now has a maximum instead of a minimum ▪ in this case, the function has been reflected in the x axis
translation	4	▪ horizontal shift ▪ affects the *x* co-ordinate of the vertex ▪ in this case, the function has shifted 4 units to the right and 4 in the *x* co-ordinate of the vertex
translation	3	▪ vertical shift ▪ affects the *y* co-ordinate of the vertex ▪ in this case, the function has shifted 3 units up and 3 is the *y* co-ordinate of the vertex

Transformations that have taken place to the basic $y = x^2$ to get $y = a(x-h)^2 + k$

transformation	variable	explanation
dilation	*a*	When *a* is greater than 1, the function is stretched vertically (*y* value increases at a faster rate than it did in the original function). When *a* is less than 1 but greater than 0, the function is compressed vertically (*y* value increases at a slower rate than it did in the original function).
reflection	*-a*	When the *a* value is negative, the function is reflected in the *x* axis, which means the function opens downwards and has a maximum value.
translation	*h*	A change in *h* causes a horizontal shift (movement on the x axis). When *h* is positive it shifts to the right and when *h* is negative it shifts to the left. The shape of the parabola does not change. A horizontal shift affects the vertex of the function and the *h* value represents the *x* co-ordinate of the vertex of the equation.
translation	*k*	A change in *k* causes a vertical shift (movement on the y axis). When *k* is positive it shifts up and when *k* is negative it shifts down. The shape of the parabola does not change. A vertical shift affects the vertex of the function and the k value represents the *y* co-ordinate of the vertex of the equation.

Part 2: Summary of Generalizations

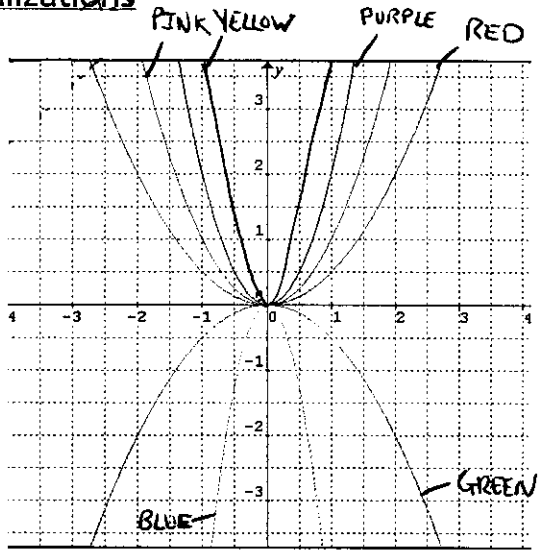
Changing the value of a :

Within quadratics, the general observations that I have made, suggests that once the value of *a* is greater, or equal to one and the numbers increase, the larger the number the narrower the opening of the parabola becomes. For example, on the diagram on the left, the red, magenta and dark blue parabolas are each wider than that of the yellow one. They each have the same vertex and *h* value, as well as the same minimum value and line of symmetry however they each have different *a* values. The yellow parabola has the highest *a* value, while the red parabola has the lowest *a* value. This proves the idea that as the a value decreases, the parabola's opening gets wider. The other name used in this case would be 'dilation'

However, in the cases where *a* is less than one and bigger than zero, the closer the number is to zero, the larger the slope of the parabola become. Again the value of *a* does not have an effect on the line of symmetry, the vertex or the range. We can see as well that the red line on the graph has a minimum value.

When the value for *a* is a negative and therefore less than zero, when the numbers decrease in value the slope of the parabola would decrease. This is seen in the graph, as the dark green line, which has a lower value, is steeper than the light blue line, which has a higher value of *a*.

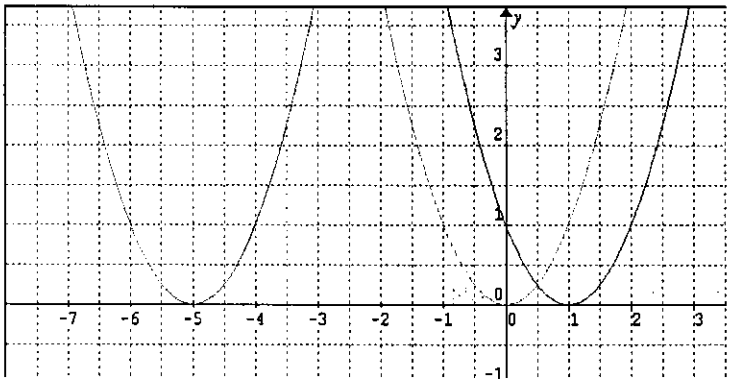
As well, when the value for *a* is less than zero, the range changes, as now the line gets a maximum value. The parabola therefore looks as if it has been flipped however, the line of symmetry and vertex don't differ from those of positive value...



(RED) $y = \frac{1}{2}x^2$ $y = x^2$ (PINK)

(GREEN) $y = -\frac{1}{2}x^2$ $y = 4x^2$ (YELLOW)

(PURPLE) $y = 2x^2$ $y = -5x^2$ (BLUE)



$y = x^2$

$y = (x-1)^2$

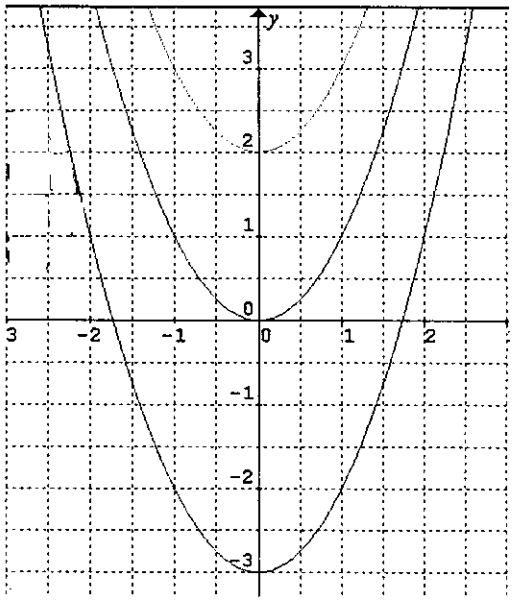
$y = (x+5)^2$

Changing the value of h :

When the value of *h* changes, it leads to the change in the line of symmetry. As well, it changes the value of the vertex. If *h* has a value that is bigger than zero, the line of symmetry will shift to the left of the x-axis, which can be shown with the red line. When h is smaller than zero the line of symmetry will shift to the right of the x-axis, as seen with the blue line. The line of symmetry becomes the value of h, with the opposite sign in front of it. This value of h, with the opposite sign in front of it, also becomes the x value for the vertex. The range is not changed and the line is always minimized. The change of *a*, does not affect the range, and the line holds a minimum value.

Changing the value for k :

When changing the value of *k*, it changes the y intercept of the line, and therefore the y value of the vertex. The *k* value is the y intercept, and therefore the bigger it is the higher up the parabola is on the graph and y axis. This is seen in the graph on the right as the pink value of the y intercept is 2 and it's y intercept becomes 2. Additionally, the value of *k* in the red line is –3, and again the y intercept becomes –3. This is as the *k* value, changes the y value in the vertex, therefore shifting the whole vertex. As the y intercept is changed, the range is changed as well. However, the line of symmetry stays the same, as the x value in the vertex is not changed.



$y = x^2$

$y = x^2 + 2$

$y = x^2 - 3$

Investigate how a, h and k affect the graph of $y = x^2$ in the equation of $y = a(x - h)^2 + k$.

Part Three: Examples to validate your general rules

Create an illustration of each of the following parabolas, using $y = x^2$ as a reference on each illustration.

Clearly explain **each step** that is required to transform the parabola $y = x^2$ into each of the following 4 separate parabolas.

For each, clearly label:

a) the axis of symmetry (equation of the line)

b) the vertex

c) the domain and range

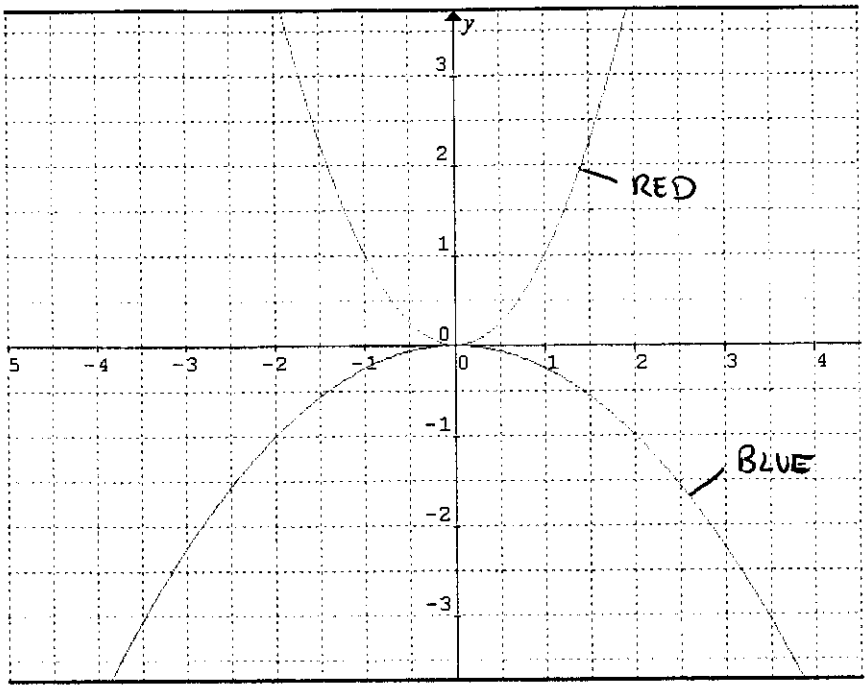
$$y = -\frac{1}{4}x^2$$

$$y = 2x^2 - 10$$

$$y = -4(x+3)^2$$

$$y = \tfrac{1}{2}(x-2)^2 + 8$$

1)



$y = x^2$ (RED)

$y = -\frac{1}{4}x^2$ (BLUE)

In order to transform the $y=x^2$ value on the graph, into the $y = -\frac{1}{4}x^2$ parabola, the *a* value of $y=ax^2$ must change. Because the value of a in the parabola $y=x^2$ is one, if changed to for example $\frac{1}{4}$, the parabola would be transformed into the mirror of the $y = -\frac{1}{4}x^2$ with the x-axis. To then put the parabola into a maximum value, we must change *a* into a negative number. When this is done, the parabola flips downwards and becomes the same line as the $y = -\frac{1}{4}x^2$.

a) the axis of symmetry (equation of the line)

The axis of symmetry is x=0

b) the vertex

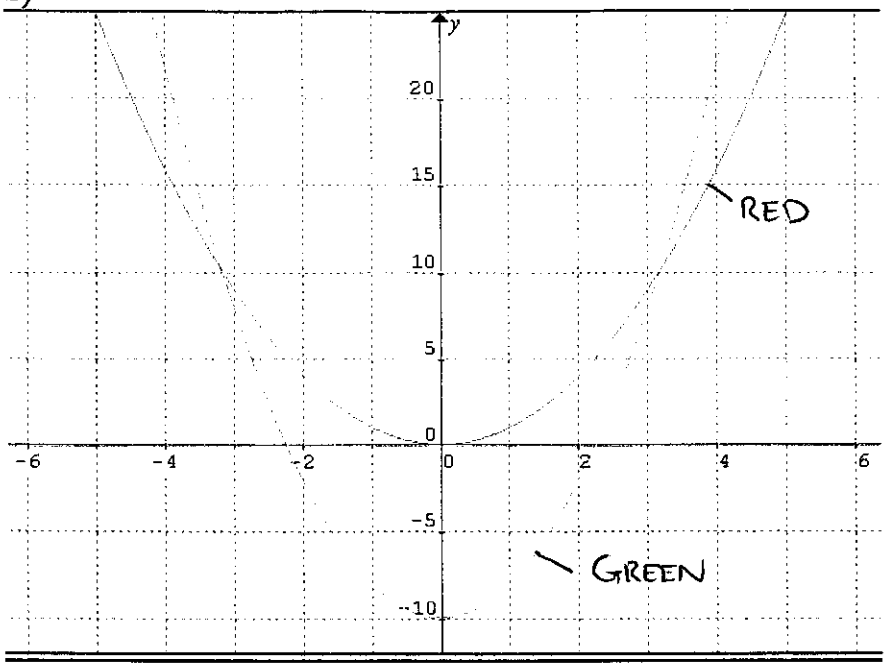
Both lines have the same axis of symmetry, as well as the same vertex which is point (0;0).

c) the domain and range

the domain of the parabola is $\{x / x \leq 0; x \in R\}$

the range of the parabola is $\{y / y \in R\}$

2)



$y = x^2$ (RED)

$y = 2x^2 - 10$ (GREEN)

In order to transform the $y=x^2$ value on the graph, into the $y = 2x^2 - 10$ parabola, the *k* and *a* value of $y=ax^2+k$ must change. Because the value of *a* in the parabola $y=x^2$ is one, if changed to for example 2, the parabola would be transformed into the same slope of the $y = 2x^2 - 10$ parabola. However this parabola would not be the same because of the *k* value. The *k* value of $y = 2x^2$ is zero, therefore in order to move the parabola down the y axis, it is necessary to change the *k* value to -10. Once this is done, the line overlaps and becomes the same line as the $y = 2x^2$.

a) the axis of symmetry (equation of the line)

The axis of symmetry is x=0

b) the vertex

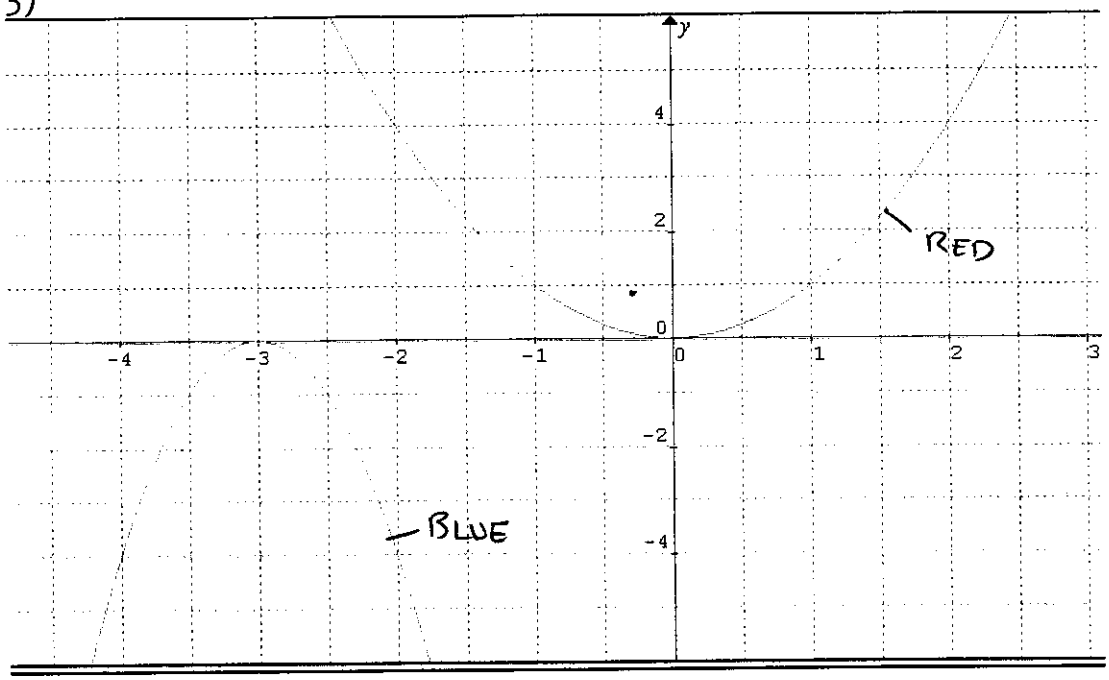
The vertex is (0;-10)

c) the domain and range

the domain of the parabola is $\{X / X \geq -10; x \in R\}$

the range of the parabola is $\{Y / Y \in R\}$

3)



$y = x^2$ (RED)

$y = -4(x+3)^2$ (BLUE)

In order to transform the $y=x^2$ value on the graph, into the $y = -4(x+3)^2$ parabola, the *h* and *a* values of $y = a(x-h)^2$ must change. Because the value of a in the parabola $y=x^2$ is one, if changed to for example -4, the parabola would be transformed into the same slope of the $y = -4(x+3)^2$ parabola. However this parabola would not be the same because of the *h* value. The *h* value of $y = -4x^2$ is zero, therefore in order to move the parabola down the x axis, towards the left, it is necessary to change the *h* value to +3. Once this is done, the lines overlap each other and becomes the same line as the $y = -4(x+3)^2$.

a) the axis of symmetry (equation of the line)

The axis of symmetry is x= -3

b) the vertex

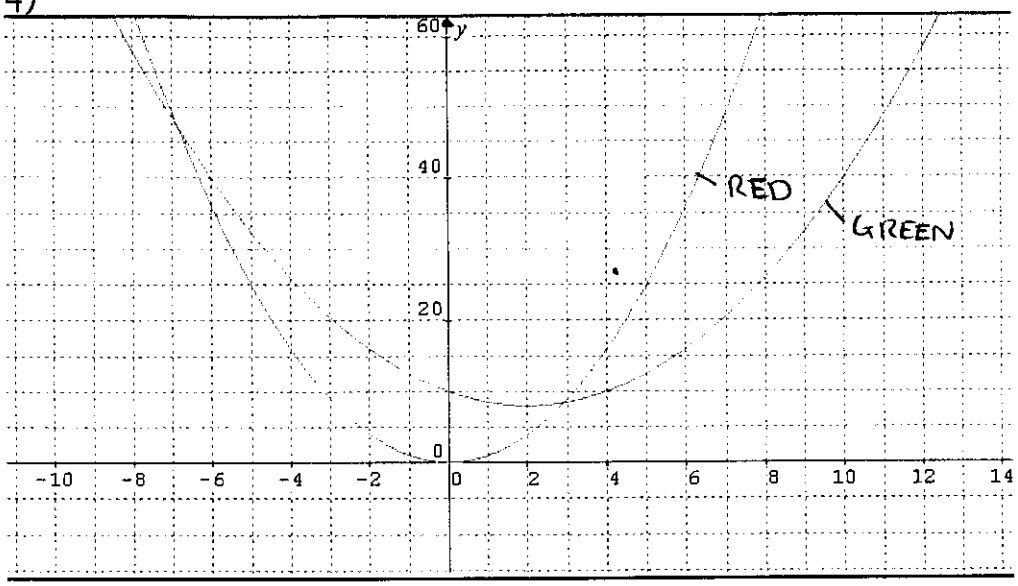
The vertex is (-3;+0)

c) the domain and range

the domain of the parabola is $\{X / X \leq 0; x \in R\}$

the range of the parabola is $\{Y / Y \in R\}$

4)



$y = x^2$ (RED)

$y = \frac{1}{2}(x-2)^2 + 8$ (GREEN)

In order to transform the $y=x^2$ value on the graph, into the $y = \frac{1}{2}(x-2)^2 + 8$ parabola, the *h*, *k* and *a* values of $y = a(x - h)^2 + k$ must change. Because the value of a in the parabola $y=x^2$ is one, if changed to for example 1/2, the parabola would be transformed into the same slope of the $y = \frac{1}{2}(x-2)^2 + 8$ parabola. However this parabola would not be the same because of the *h* value.

The *h* value of $y = \frac{1}{2}x^2$ is zero, therefore in order to move the parabola down the x axis, towards the right, it is necessary to change the *h* value to -2. The *k* value must also be changed in order for the parabola to move up the y axis, to +8 Once this is done, the lines overlap each other and becomes the same line as the $y = \frac{1}{2}(x-2)^2 + 8$.

a) the axis of symmetry (equation of the line)

The axis of symmetry is x= 2

b) the vertex

The vertex is (2;1.7)

c) the domain and range

the domain of the parabola is $\{x / x \geq 1.7 : x \in R\}$

the range of the parabola is $\{y / y \in R\}$

Part Four: Reasoning and Justification

Answer the following questions in sentence form, using examples where suitable to illustrate your point:

How did the use of technology help you during the course of this investigation?

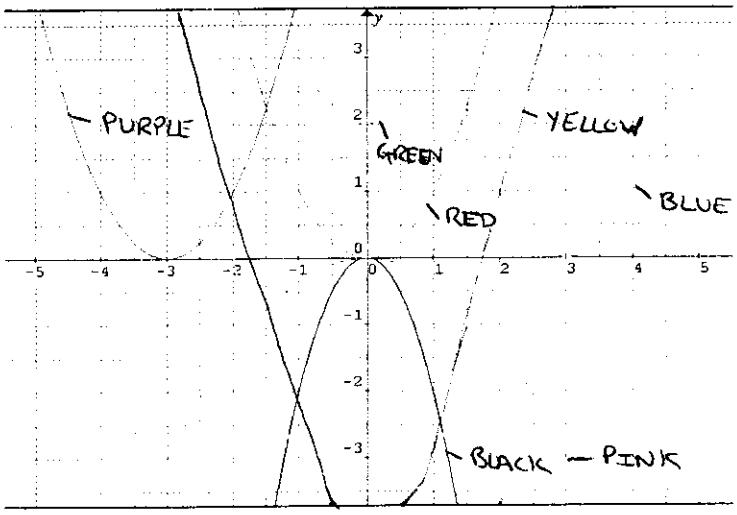
The use of technology helped me during this investigation because it helped to visualize the graph, rather than depending on my knowledge of parabolas and the *a*, *h* and *k* values and how they affect quadratics. I also used math software such as Graphmatica and Math type. They helped me during this investigation because they made it much easier to display.

Can you think of any improvements that you could have made to your problem solving methodology during this assignment?

A way that I could have improved my problem solving methodology would be to validate my calculations and answers however I did not have enough time. Another way of improving my method would be to use as many examples as possible to fully display and demonstrate my outlook.

Why is the generalized form of the equation expressed as $y = a(x-h)^2 + k$?

With the $y = a(x-h)^2 + k$ equation, each coefficient *a*, *k* and *h* have individual effects on the parabola. When each is altered or changed, it is clearly changed on the actual parabola. This can be seen with the red parabola $y=x^2$. It is the only equation that has values such as 0 or 1 for the *a*, *h* and *k* variables. Obviously, as viewed on the graph, the characteristics of each parabola change as the variables change.



$y = x^2$ (RED)

$y = \frac{1}{3}x^2$ (PINK)

$y = 2x^2$ (BLACK)

$y = (x+3)^2$ (PURPLE)

$y = (x-3)^2$ (BLUE)

$y = x^2 + 2$ (GREEN)

$y = x^2 - 4$ (YELLOW)

How do you think this formula could be used in real life applications?

A real life example of quadratics and a parabola would be a bridge. This formula could help to measure bridges, etc. With the values of *a*, *h* and *k*, it would be possible to determine characteristics such as the line of symmetry (the centre/middle of the bridge) or the vertex (which would be the height of the bridge).

How do you know your generalizations about transformations are correct?

Justify/validate your general rules.

I know that my generalizations are correct based on my observations of how the parabola on the graph changes, just as the equation changes. For example, when observing the parabolas on the graph above, the red, dark blue and cyan parabolas all have their vertexes on the x-axis. By observing the equations, I am able to see that it is the value of the *h* coordinate which has an effect on how the parabola changes on the graph.

Can these generalizations be applied to other functions? Explain your reasoning.

I don't believe that this generalization can be applied to other functions. I looked at other quadratic equations that had the particular *a*, h and *k* coefficients.

Moderation checklist

Check for:

- number of judgments per criterion
- minimum tasks
- background information.

Background information:

- mark schemes
- copy of worksheet or test paper
- descriptions of the ways the assessment tasks were presented to the student
- any relevant materials that could help in the moderation process.

The background information should be contained in an additional folder. Background information should always be a part of a sample.

Superscript refers to the number of judgments against each criterion.

Mathematics—A^2 B^2 C^2 D^2

(Standard mathematics and extended mathematics)

Prescribed minimum:

- broad based classroom test/examination (covers at least three of the branches of the mathematics framework)
- A mathematical investigation (completed under test conditions)
- A real-life problem.

see *Mathematics guide (2007)* p46-49

	Middle Years Programme	Form F3.1
	Moderation coversheet: Subjects	

Please complete a copy of this form for **each** folder of work submitted for moderation.

Please ensure that the material being submitted for moderation conforms to the requirements set out in the relevant subject group guide. All the criteria **must be applied twice** within the folder accompanying this form, unless stated otherwise in the subject guide.

School name: *Treetops International school* School code: *1003*

Student's name/number: *James White* Subject: *Extended mathematics*

The student's work is (please mark box):

☐ comparatively good ☒ *x* average ☐ comparatively weak

Nature and title of assessment task		Criteria					
		A	B	C	D	E	F
1. *Broad based test year 4*	Teacher	*6*					
	Moderator						
2. *Investigation: Cutting corners*	Teacher		*5*	*5*			
	Moderator						
3. *Real-life problem: Medicine and mathematics*	Teacher	*4*		*4*	*5*		
	Moderator						
4. *Investigation: Egyptian triangulation*	Teacher		*7*		*3*		
	Moderator						
5.	Teacher						
	Moderator						
6.	Teacher						
	Moderator						

Please use the reverse of this form or separate sheets to identify the conditions under which each piece of work was done (project, classroom test, end-of-term examination and so on), the amount of support provided, any special circumstances, and general/specific information on the student. Provide any information that may assist the moderators in determining how the criteria were applied.

Name of teacher: Marie Anne Williams

Signature of teacher: *MA Williams* Date: *17 July 2008*

Names of teachers involved in internal standardization for this subject:

Susan Roberts and Liu Hua

Teacher's comments:

Real life problem: Medicine and mathematics

James was ill during the week the task was set and carried out. He carried it out a week later in school during extracurricular time.